# ОПРЕДЕЛЯНЕ НА ЗАКОНА ЗА УПРАВЛЕНИЕ НА ОПАШНИЯТ МЕХАНИЗЪМ НА ТРИКОПТЕР

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# DEFINING THE CONTROL LAW FOR YAW MECHANISM CONTROL OF A TRICOPTER

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***Abstract:*** *Behavior of a specific constructive scheme of a tricopter is modeled. On the base of real characteristics of propeller-motor system, a model of the motion of the yaw mechanism is worked out. A flight stabilization method is proposed.*

**Keywords**: TRICOPTER, YAW MECHANISM, FLIGHT STABILIZATION

## *1. Introduction*

Copters are drones, which are capable of flying due to the lift created by the propellers. There are different design solution such as tricopters, quadrocopters, etc. Some of the possible design solutions are shown on fig.1.

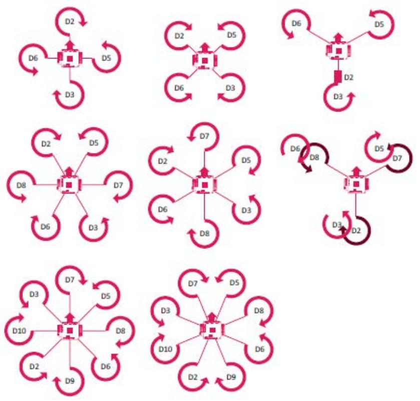


***Fig. 1*** *Copters design solution.*

The main problem with this type of unmanned aircraft is the existence of reactive moment, created by the rotors. If the copter has an even number of engines, torque is counterbalanced by opposite rotating propellers. If there is an odd number of engines, torque is countered by tilting one of the engines, thus creating a moment counteracting the reactive moment in the XOZ plane of the Body Frame.

For the purpose of this research, a tricopter scheme was selected, where the following alternatives are available:

1. Tricopter with an even number of rotors – 4 or 6(fig.2)
2. Tricopter with an odd number of rotors – 3(fig.3)

***Fig. 2*** *Tricopters with even number of rotors.*

A tricopter with an odd number of engines – three, was created. There are two possible design solutions - with unidirectional and mixed rotation of the front propeller. However, the problem with the compensation of torque remains. In order to keep the design at a low cost, the one with the unidirectional rotation of the propellers was chosen.(fig. 3)

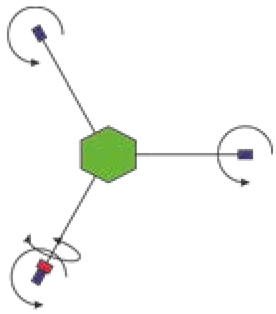


***Fig. 3****. Tricopter with an odd number of rotors*

## *2. Theoretical and experimental researches*

The syntheses of the tricopter control algorithms, requires a mathematical model of the vehicle.

In order to derive the expressions for the forces and moments acting on the aerial vehicle, the following assumptions were made:

- The aircraft is a rigid body.
- The mass of the aircraft is symmetrically distributed.

From the definition of body frame it follows that the axes coincide with the major inertia axes of the aircraft (fig.4). Following these assumptions, the movement of the aircraft is described as a movement of a rigid body with 6 DOF.

To accurately describe the motion of a solid body with 6 DOF the Euler equations are used. They can be derived from the famous amount of movement alteration and kinetic moment alteration theorems. Thus, the free movement of the rigid body may be split into two movements:

- translational movement of the center of mass.
- rotation around the center of mass.

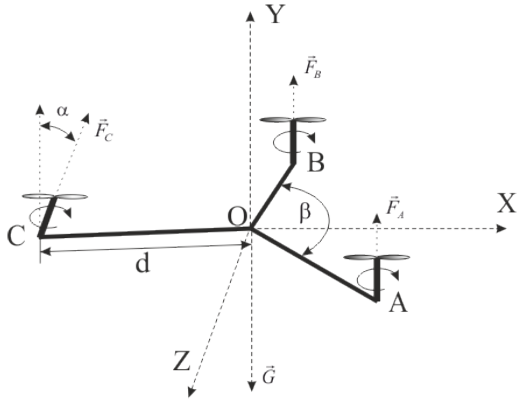



***Fig. 4*** *Modeling the movement of a tricopter.*

The equations for translational movement are derived from the amount of movement alteration theorem.

$$m\frac{d\vec{V}}{dt} = \sum \vec{F} \qquad (1)$$

where: $\frac{d\vec{V}}{dt}$ is the absolute acceleration;

$\sum \vec{F} = \vec{R} + \vec{G}$ is the resultant of the external forces;

$m$ is the mass of the tricopter.

Since the relevant forces and moments are presented in a coordinate system Oxyz (фиг.4), it is convenient for the absolute acceleration $\frac{d\vec{V}}{dt}$ to be expressed through its projections in Oxyz: $\frac{\tilde{d}\vec{V}}{dt}$. Body frame performs rotation around the center of mass with angular velocity and so the absolute acceleration is defined as follows:

$$\frac{d\vec{V}}{dt} = \frac{\tilde{d}\vec{V}}{dt} + \vec{\omega} \times \vec{V} \qquad (2)$$

Rotation around the center of mass is described by the kinetic moment alteration theorem.

$$\frac{d\vec{K}}{dt} = \sum \vec{M} \qquad (3)$$

where: $\sum \vec{M}$ is the resultant vector of the external moments;

$\vec{K}$ is a vector of the kinetic moment, that is determined by the ratio:

$$\vec{K} = \sum\left(\vec{r}_k \times m_k \vec{V}_k\right) \qquad (4)$$

where $r_k$ is the distance from the center of gravity to each point of the body; $m_k$ is the mass of each point of the body; $V_k$ is the linear velocity of any point of the body;

In reference to fig.4 the following equations describing the motion of the tricopter are derived:

1. For the balance of the forces:

$$\begin{aligned} &\sum F_x = -G.\sin(\vartheta); \\ &\sum F_y = F_A + F_B + F_C.\cos(\alpha) - G.\cos(\gamma).\cos(\vartheta); \\ &\sum F_z = F_C.\sin(\alpha) + G.\sin(\gamma).\cos(\vartheta). \end{aligned} \qquad (5)$$

2. For the balance of the moments:

$$\begin{aligned} &\sum M_x = (F_A - F_B).d; \\ &\sum M_y = F_C.\sin(\alpha).d - \left(M_{A_{yp}} f(F_A) + M_{B_{yp}} f(F_B) + M_{C_{yp}} f(F_C, \cos(\alpha))\right); \\ &\sum M_z = (F_A + F_B).d.\cos\left(\beta/2\right) - F_C.\cos(\alpha).d + M_{C_{zp}} f(F_C.\sin(\alpha)). \end{aligned} \qquad (6)$$

Taking into account expressions (5) and (6) and substituting them into expressions (1) and (3) we can obtain the differential equations of the motion of the tricopter.

To determine the position of tricopter in relation to the stationary coordinate system, the three Euler equations defining the relationship between angular velocities and angles of roll (γ), pitch (υ) and sliding (ψ) are used. The coordinates of its center of mass are obtained using the kinematic equations linking the derivative of the coordinates of this center with the projections of its speed and angles of roll, pitch and sliding.

In solving the equations of motion of the tricopter, for the forces of thrust $(F_A, F_B, F_C)$ and the reactive moment of the propellers $(M_{A_p}, M_{B_p}, M_{C_p})$ are used the results obtained from [2]. On fig.5 and fig.6 are presented the graphics illustrating the character of the change of the thrust and the torque for a single motor-propeller system.

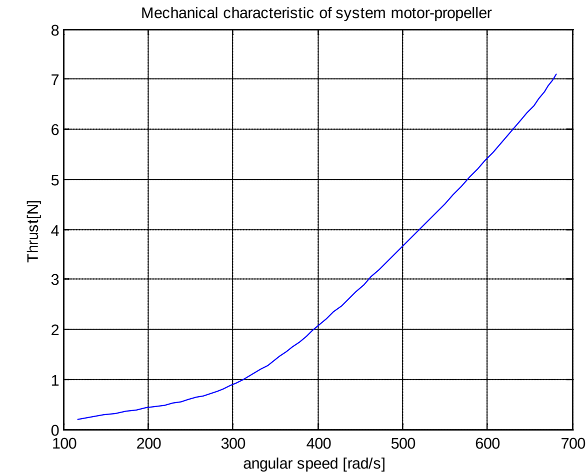



***Fig. 5*** *Nature of the motor -propeller system thrust change.*

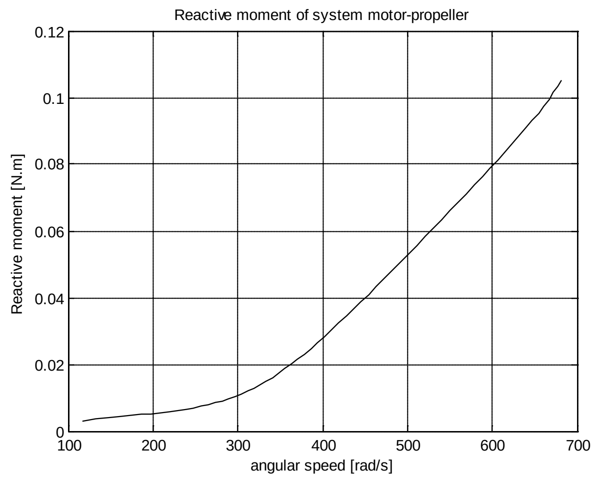



***Fig. 6*** *Nature of the motor-propeller system torque change.*

The displayed graphs of the thrust and torque of an engine-propeller system are dependent on the rotational speed of the propeller.

According to expression (6) for balancing the tricopter in the horizontal plane $M_y=0$. To offset the resultant reactive moment of the three blades, it is necessary for the tail mechanism of the tricopter to be rotated on a certain angle (α) depending on the current rotational speed of the blades. The balancing curve of the variation of the angle of the tail plane when the tricopter is hovering is shown on fig.7.

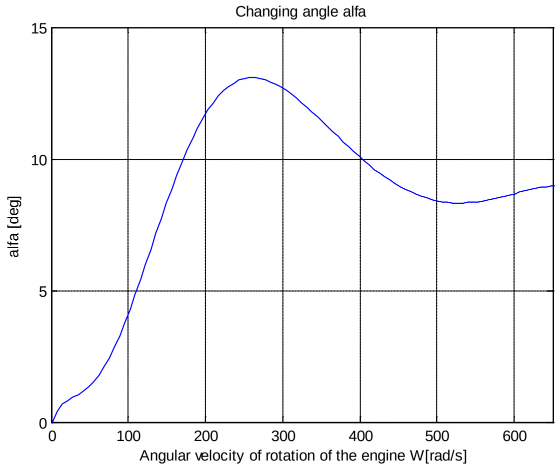



***Fig. 7** Modification of the angle of the tail assembly as a function of the angular speed of rotation of the blades.*

Using the complete mathematical model for such a body, a case is studied in which the tricopter hovers in a selected point above the surface of the Earth. The tricopter control is carried out by tilting the tail rotor so as to achieve the desired angle $\psi_{зад}$ in the horizontal plane.

In this case, the nonlinear mathematical model in expressions (1) and (3) becomes (7).

$$\frac{d\omega_y}{dt}=\frac{\sum M_y}{I_y};\quad \frac{d\omega_z}{dt}=\frac{\sum M_z}{I_z};\quad \frac{d\psi}{dt}=\omega_y. \tag{7}$$

So the nonlinear mathematical model expressed with differential equations (7) is linearized using the method of partial derivatives in the previously selected state vector $x=[\omega_y,\omega_z,\psi]$ and vector of control $u=[\omega_A,\omega_B,\omega_C,\alpha]$.

The vector of control incorporates the angular rotational speeds of the front propellers, the rear propeller and the angular deviation of the tail mechanism of the tricopter. The output y is drawn from the angular velocities along the axes Y and Z, as well as the angle of rotation in the horizontal plane $\psi$ .

After linearization of the nonlinear model in expression (7), a model of the system in the state space is obtained– expression (8).

$$\dot{x}=Ax+Bu;\quad y=Cx+Du. \tag{8}$$

The open system obtained following the model is on the border of stability.

Of the many options for setting the coefficients of the matrix of feedback K a linear quadratic regulator (LQR) is selected. From the theory it is known that LQR indirectly gives account of the response of the system, the limits of the amplitude of controllable variables and the controlling impacts.

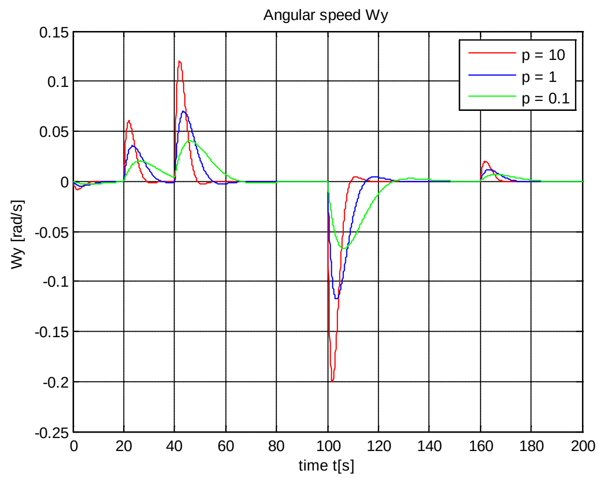
Determining a reference point for the linearization of the model is carried out based on the selected mode of hovering. In this case the following requirements have to be fulfilled - $\sum F_y=0$, $\sum M_y=0$ and $\sum M_z=0$. From here shall be identified base values of the vector of control.

A specific feature of the use of LQR is determination of the weight matrices Q and R.

$$J=\int_0^\infty [x^T(t)Q\,x(t)+u^T(t)R\,u(t)\,dt] \tag{9}$$

These matrices need to be positively determined. One way to initially set these matrices is as diagonal matrices with elements in the main diagonal with values equal to $1/(x_{imax})^2$ and $1/(u_{imax})^2$. In this case $x_{imax}$ is the maximum variation of the corresponding element in the state vector, and $u_{imax}$ is the maximum variation of the corresponding element of the vector of control.

After closing the system with a regulator-defined optimal matrix K, the results shown in fig.8, fig.9 and fig.10 are obtained.

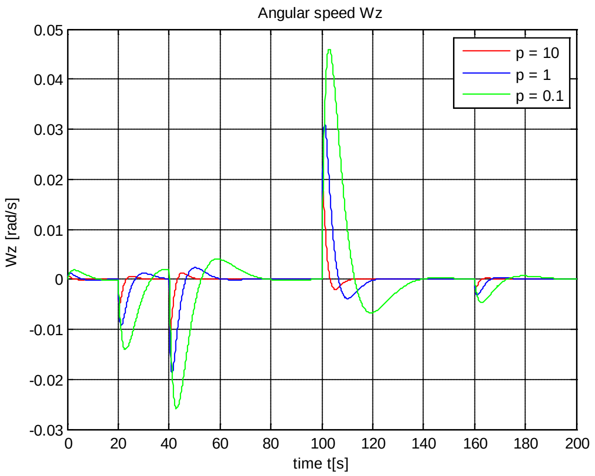



***Фиг. 8** Alteration of the angular velocity $\omega_y$ .*

Fig.8 shows alteration of angular velocity $\omega_y$ depending on the control applied. A family of characteristics is obtained at different values of the weight factor p, expression (10).

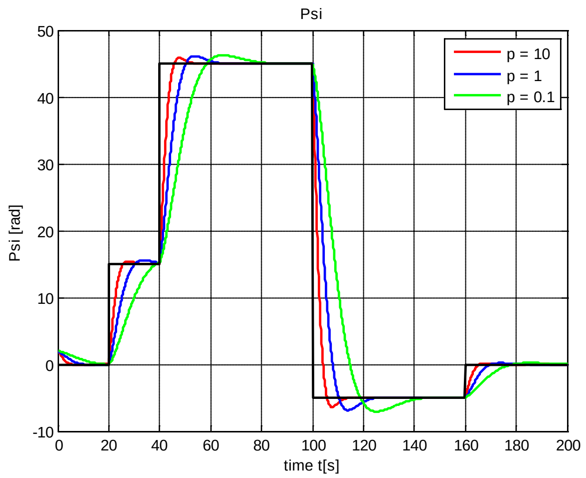
$$J=\int_0^\infty [x^T(t)\,p\,Q\,x(t)+u^T(t)R\,u(t)\,dt] \tag{10}$$

The next figure (fig.9) shows the change in the angular velocity of the tricopter along axis Z.




***Фиг. 9*** *Alteration of the angular velocity $\omega_z$ .*

Again, there are obtained a family of characteristics with different weight factor p.




***Фиг. 10*** *Altering the angle $\psi$ depending on the control applied.*

In fig.10 is shown a family of characteristics, obtained at different weight factor p, of the angle of rotation in the horizontal plane of the tricopter. In the simulation, the desired control is supplied in the form of a step function (fig.10), a transitional process of the system is represented through a family of baseline characteristics.

A change of the control signal ($\psi_{зад}$) in both directions (↕), leads to a change in angular velocities along the axes У and Z, which is compensated by the regulator. The tendencies in the change of the variables in the simulations of the mathematical model correspond to the behavior of the real tricopter (fig.11).

### *3. Conclusions and results*

1. The selected tricopter design requires a greater angle of alteration of the tail rotor to offset the resultant reactive moment;
2. The balancing angle of alteration of the tail rotor (α) was calculated to provide zero angular velocities;
3. The resulting matrix K is optimal, but due to the fact that a consistent theory for the initial determination of the weight matrices Q and R is not available, the behavior of the system with three different weight factors is simulated;
4. With the increase of p, the alteration of the angle $\psi$ approaches the desired values, but this results in increased $\omega_y$ . Theoretically, it was found that when p=10 resulting angle speeds are safe for the tricopter;
5. An optimal control system of the rotation angle in the horizontal plane was proposed.

***Fig. 11*** *Flight of the created trikopter.*

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