SUGGESTION OF SPARE PARTS INVENTORY THEORY MODEL IN ARMY

Assoc. prof. Eng. J. Furch, Ph.D.
Faculty of Military Technology - University of Defence - Brno, the Czech Republic,
E-mail: jan.furch@unob.cz

Abstract: *The article introduces a model which might be used for supplying spare parts of ground forces technical equipment to the Army of the Czech Republic. When designing inventory management, an Economic Order Quantity (EOQ) model is applied. It is a classical model used to determine stock order quantity. When setting the model, stock classification based on the ABC method and the EW matrix is made. Following the stock classification, for the Army of the Czech Republic we would suggest determining a signal stock level which is the same as for the Q-model of a stock management system. In this model used to optimize the stock we apply a dynamic model with absolutely determined stock movement, a dynamic multiproduct model where purchase cost is constant, and a dynamic model completed by a temporary unsatisfied demand. The important aspects which should be taken into consideration when setting an inventory model are stock purchase cost, storage costs, stock holding cost, or the cost which arise due to a short supply.*

KEYWORDS: *ECONOMIC ORDER QUANTITY, INVENTORY THEORY MODEL, SPARE PARTS, P AND Q – SYSTEM OF SUPPLY CONTROL.*

1. Introduction

Inventory theory might be described as a set of mathematical methods used to model and optimize the processes of generated items to ensure smooth company running. When determining the strategy under certainty conditions, it is necessary to asses and offset ordering cost against stock holding cost. The strategy to order a large amount less frequently might often result in inventory holding cost increase which can be higher than ordering cost savings. The ordering cost consists of settling stock transfer, product handling cost, product storage costs, and settling an invoice cost. The inventory holding cost consists of the capital tied up in stocks cost, service cost (insurance, taxes), storage facilities cost, and also the cost of stock deterioration risk.

When setting an order strategy the aim of which is to minimize total inventory holding costs and ordering costs, it is possible to use **Economic order quantity (EOQ)** [1]. The model EOQ is a concept which determines optimal order quantity based on ordering costs and holding costs. The optimal order quantity is when incremental ordering costs equal incremental holding costs. Cost relations which are to be taken into account when setting economic, i.e. optimal order quantity.

One of the basic characteristics in stock management models is the nature of the observed stock unit demand. This demand might be described by different models which can be divided into two basic criteria:

1. When taking into account the way of determining an enquiry (consumption) level and lead time length, there are:
   a) deterministic models which assume the enquiry (consumption) quantity and lead time length to be precisely known,
      - Q - system of supply control – the delivery frequency changes while the delivery size is constant,
      - P - system of supply control – the delivery size changes while the delivery interval is fixed.
   b) stochastic models based on the probabilistic character of enquiry (consumption) and the length of lead time.
   c) non-deterministic models where the character of enquiry (consumption) and lead time is not known.
2. When dealing with the way of renewing supplies, there are:
   a) static models where the supply is made by one-time delivery,
   b) dynamic models where an item supply is kept in stock on a long term basis and is renewed by frequent deliveries.

The supply theory models introduced above use a basic evaluation criterion based on minimizing the overall costs of purchasing, storing and keeping supplies, and in some cases even short supply costs.

2. Results of discussion - Design of Dynamic Inventory Theory Model with Determined Movement of Spare Parts

In order to calculate inventory theory optimization within the conditions of the Army of the Czech Republic, we would suggest using a dynamic model with absolutely determined spare parts movement for the orders of separate spare parts, a dynamic multiproduct model with a constant amount of cost including supply purchase for the associated orders of spare parts, and a dynamic model completed by a temporary unsatisfied demand.

2.1 Proposal of a Model for Controlling Spare Parts Vehicles Stock

The model proposal is based on the EW matrix which includes the ABCD analysis completed by the XYZZ′ analysis. The ABCD analysis will be used for classifying spare parts by a material price during purchase. The group A stands for the material of the highest price and the group D on the other hand means that the material is of the lowest purchase price. Next we will classify the material into XYZZ′ groups by stock rotation per year. The group X means that spare parts are of the highest rotation. On the other hand the group Z′ stands for the spare parts with zero rotation. Along with these two criteria we select one more criterion which expresses delivery time of a spare part. The marking I stands for a short delivery time of spare parts, while the group labelled as IV means that the delivery time of spare parts is the longest.

On the basis of monitoring and then analyzing the criteria we can classify the assortment of spare parts into 64 groups. Next, the stock rotation of single spare parts and presumed term of delivery are determined. After the spare parts classification according to the described model is performed, a signal level amount will be calculated for each group at a certain risk level in terms of early delivery and determination of delivery optimum amount. This risk is to be considered in the light of purchase cost and spare parts stocking, and that is the main criterion when it comes to selecting a proper model. In the military area it is necessary to take into account the coefficient of vehicle technical availability which serves as input information for determining a safety stock level. This is most closely related to the calculation of stock provision reliability, see Chapter 2.6.

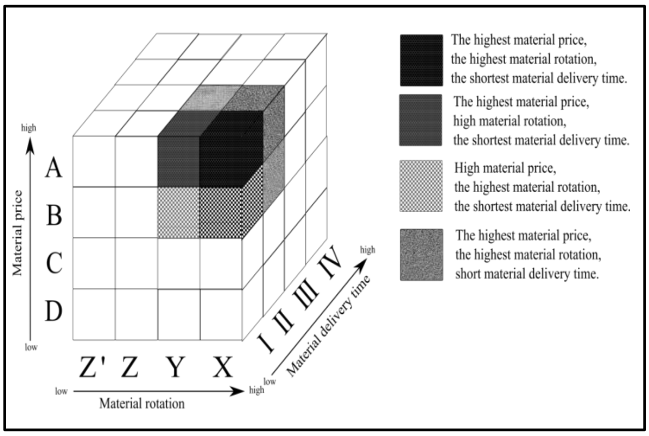

Fig. 1 *Proposal of Economic Wizard Matrix for 64 groups of vehicles spare parts*

2.2 Selected methods dealing with prediction of spare parts demand

Next step when setting optimal spare parts stock control is to forecast the future consumption of items in stock. The forecast is based on the consumption history which should be representative, or, in other words, long enough. In case of spare parts we regularly work with the history of five to ten years, in case of occasional items it is at least three years of registered consumption history (generally the following rule applies: the longer the history the more accurate and reliable the forecast). The methods used to calculate:

a) Moving Average Method.
b) Moving Weighted Average Method.
c) Least Squares Method.
d) Brown´s Method.
e) Holt-Winters Method.
f) Smart-Willemain Method.
g) Smart-Willemain Method.

In 2002 Smart and Willemain introduced a simulation statistical method which addresses the stochastic prediction of future consumption. With this method it is possible to set a minimum stock level (a reorder level) so that the demands could be met with a determined probability (logistics service). The Smart-Willlemain method is based on random sampling from a consumption history (in statistics this procedure is called bootstrapping).

2.3 Dynamic model with absolutely determined stock movement

It is a model which assumes that the quantity of enquiry is exactly known in advance. The aim of the optimization is to determine an optimal delivery size $x_{opt.}$ for which the total costs connected with delivery acquisition, keeping and storing supplies $N_c(x_{opt.})$ during the period of a length T will be minimal.

If it is necessary to purchase Q units during the period of a length T and the supply is regularly renewed by the delivery of x units, then the number of deliveries v per a given period might be expressed by the following formula [1]

$$n = \frac{Q}{x}\,. \tag{1}$$

The costs used for acquiring all deliveries $N_p(x)$ are given by the product of a delivery amount n and one delivery costs c_p [1]

$$N_p(x) = nc_p = \frac{Q}{x}c_p\,. \tag{2}$$

The costs used for keeping and storing supplies $N_s(x)$ during a period T depend on the quantity of average supply x. An average stock level $\bar{x}$ equals exactly the half of a delivery size x. The storing costs can be put as follows [1]

$$N_s(x) = \frac{x}{2}Tc_s \tag{3}$$

where c_s is the cost which includes maintaining and storekeeping per one order.

Along with the increase in delivery size, an average carry over supply and consequently the total costs of keeping and storing supplies grow too. By summing both cost items we obtain a total cost function $N_c(x)$ which might be expressed in the following manner [1]

$$N_c(x) = \frac{Q}{x}c_p + \frac{x}{2}Tc_s\,. \tag{4}$$

Now we set the first derivative of formula (7) equal zero and we get the Harris-Wilson equation which is put this way

$$x_{(opt.)} = \sqrt{\frac{2Qc_p}{Tc_s}}\,. \tag{5}$$

When we put an optimum delivery size expression $x_{opt.}$ in a cost function $N_c(x)$, we will get the formula used for calculating minimum total costs

$$N_c(x_{opt.}) = \sqrt{2QTc_pc_s}\,. \tag{6}$$

Optimum length of a re-order cycle $t_{(opt.)}$ might be expressed by the formula [1],

$$t_{(opt.)} = \frac{Tx_{(opt.)}}{Q} = \sqrt{\frac{2Tc_p}{Qc_s}}\,. \tag{7}$$

Besides total minimal so-called optimum costs it is necessary to determine the time when an order is to be raised to get a new order to the store in time. Therefore it is essential to set an optimum so-called ordering supply level r_0

$$r_o = Qt_p - mx_{opt.} \tag{8}$$

where: t_p – the length of order lead time, t_c – the length of a delivery cycle, m – the amount of orders a route.
The value m will be calculated the following way

$$m = \frac{t_p}{t_c}\,. \tag{9}$$

The calculated value m will be rounded up to whole numbers.

<u>When following this way, the requirements listed below are to be observed</u> [1]:

- enquiry (consumption) should be known and constant,
- supply collection cannot swing,
- supply renewal is one-time in the form of an optimum delivery size,
- acquiring and storing costs are to be stabilized,
- purchase price is independent from order quantity,
- optimum delivery size is calculated for each supply item separately.

2.4 Dynamic multiproduct model with constant stock purchase

This model assumes the customer will order more items to a store at once. On that account, however, an optimum delivery cycle as well as the optimum delivery size of single items will be abandoned, which can affect the costs of supply keeping and storing. This model presumes that supply acquisition costs do not depend on the number of ordered items. Furthermore, it is assumed that for the length T period the army needs to order k items of supplies with expected consumption (enquiry) Q_i of quantity units. The costs of the keeping and storing of single supply items are c_{si}. The total costs of a group order are then expressed by formula (13), provided that the items will be delivered in equal length delivery cycles t_c. The number of deliveries n is to be also the same [3]

$$N_c(t_c) = \frac{T}{t_c}c_p + \frac{1}{2}t_c\sum_{i=1}^{k}Q_ic_{si}\,. \tag{10}$$

If a function $N_c(t_c)$ derivative by t_c equals zero, then for an optimum delivery cycle length we will obtain the formula below [3]

$$t_c^{opt.} = \sqrt{\frac{2Tc_p}{\sum_{i=1}^{k} Q_i c_{si}}} . \quad (11)$$

Now it is necessary to determine an optimum structure (amount) of single material items delivery [3]

$$x_i^{opt.} = \frac{Q_i t_c^{opt.}}{T} . \quad (12)$$

After, we will specify the total minimal costs of supply acquisition for item group ordering in the following manner [3]

$$N_c(t_c^{opt.}) = \sqrt{2Tc_p \sum_{i=1}^{k} Q_i c_{si}} . \quad (13)$$

Beside the total minimal optimum cost we should also set the time when to raise an order so that the new order could arrive at the storehouse in time. Therefore it is necessary to determine an optimum stock signal level.

$$x_{o_i} = Q_i t_p - m x_i^{opt.} \quad (14)$$

where: t_p – the length of order lead time, t_c – the length of a delivery cycle, m – the amount of orders a route.
The value m will be calculated the following way

$$m = \frac{t_p}{t_c} . \quad (15)$$

2.5 Dynamic model completed by temporary waiting demand

In the dynamic model let us presume that there is a temporary lack of inventory on hand. It means that the demand for particular items might be temporarily unsatisfied. Therefore the re-order cycle splits into two intervals. During the first interval there is a withdrawal form inventory and the withdrawal time is designated as t_1. During the second interval there has been the lack of inventory on hand and the demands for a withdrawal form inventory which occur during this interval are not satisfied. The interval length is denoted by t_2. The length of a delivery cycle is then

$$t = t_c = t_1 + t_2 . \quad (16)$$

The level of waiting demand during the interval t_2 is marked with s. This model presumes that this waiting demand will be satisfied immediately after the delivery is in stock. Out of the total supply portion x s items will be immediately used for satisfying waiting demands and the rest in amount of (x-s) items will be placed in stock. The maximum inventory held level might be then only (x-s).

The total cost of storage and material acquisition will also include short supply cost N_z. It consists of inventory carrying cost N_{snz}, acquisition cost N_{pnz} and short supply cost N_z. The inventory carrying cost within one delivery cycle might be expressed as a product of average stock the size of which is in each cycle $(x\text{-}s)/2$, inventory carrying cost per unit c_s, and the time t_1, during which the stock is being withdrawn [4]

$$N_{s_{nz}} = c_s \frac{x-s}{2} t_1 . \quad (17)$$

In each cycle at least one delivery, which is related to acquisition cost N_{pnz} amounting of unit cost c_p, is acquired. Short supply cost N_z is calculated within one cycle as a product of mean short supply, i.e. $s/2$ of unit cost c_{nz} and the time t_2 during which the stock is not available [4]

$$N_z = c_{nz} \frac{s}{2} t_2 . \quad (18)$$

Within one cycle the total cost equals the sum of the three given items. During an observed period (one year for example), however, Q/x cycles pass, where Q is the total yearly demand size when temporary short supply occurs, and x is the size of one delivery during temporary short supply. In order to calculate total cost during all period it is sufficient to multiply one cycle short supply cost by the number of cycles. The resulting cost function will be then as follows

$$N_{c_{nz}}(x,s) = \left(c_s \frac{x-s}{2} t_1 + c_p + c_{nz} \frac{s}{2} t_2 \right) \frac{Q}{x} . \quad (19)$$

This is a function of two variables x and s. In the above formula there are besides variables x and s also time characteristics t_1 and t_2. After some changes the resulting equation is as follows [4]

$$N_{c_{nz}}(x,s) = c_s T \frac{(x-s)^2}{2x} + c_p \frac{Q}{x} + c_{nz} T \frac{s^2}{2x} . \quad (20)$$

The extreme value of the function will be calculated so that the partial derivative by variables x and s equals zero. The optimum values of delivery size $x_{opt.}$ and the optimum waiting demand size s_0 are the outcome of the above calculation:

$$x_{opt.} = \sqrt{\frac{2Qc_p}{Tc_s}} \sqrt{\frac{c_s + c_p}{c_{nz}}} , \quad (21)$$

$$s_0 = \sqrt{\frac{2Qc_p}{Tc_s}} \frac{c_{nz}}{c_s + c_{nz}} . \quad (22)$$

In the temporary short supply model providing the demand is not satisfied, the optimum order size will increase, but at the same time the mean stock size will decrease.

The optimum length of a delivery cycle t_0 corresponding to the optimum order size $x_{opt.}$ is calculated in the following way [4]

$$t_0 = \sqrt{\frac{2c_p T}{c_s Q}} \sqrt{\frac{c_s + c_{nz}}{c_{nz}}} \quad (23)$$

Minimum achievable cost N will be obtained by putting expressions (24) and (25) into the cost function (23)

$$N_{\min}(x_{opt.}, s_o) = \sqrt{2QTc_s c_p \sqrt{\frac{c_{nz}}{c_s + c_{nz}}}} . \quad (24)$$

The optimum signal supply level x_0 will be calculated so that we subtract the size of en route order and the number of temporary waiting demands from the expected demand during the time t_p. [4]

$$x_0 = Qt_p - m x_{opt.} - (x_{opt.} - s_o) . \quad (25)$$

2.6 Calculation of supply ensuring reliability

The reliability of ensuring supply is about the way to protect the company against short supply with safety stock. In practice this is assessed by a service level or level of stock availability. However, in both cases it remains true that the higher the level of ensuring reliability, the higher the level of safety stock which grows over-proportionally.

Service level α expresses the probability (t_1/t) that within one delivery cycle no short supply occurs. Logical complement (1-α) expresses the probability that the customer demand will not be satisfied. The level of stock availability β might be defined as the probability (t_2/t) that the order can be fully satisfied immediately after it is exercised from the stock. The complement (1-β) expresses what relative part of the total demand will not be satisfied when short supply occurs. The values α, β might be expressed by the following equations [5]:

$$\beta = \frac{s_0}{x_{opt.}} = \frac{c_s}{c_p + c_{nz}}, \quad (26)$$

$$\alpha = 1 - \beta = \frac{c_{nz}}{c_s + c_{nz}}. \quad (27)$$

2.7 Setting safety stock level

Determining the safety stock level is most often based on normal distribution of random variables of demand (consumption), deliveries and lead time. However, the precondition of normality is not always fulfilled in practice. Therefore it is advisable to perform a relevant test, e.g. the chi square goodness of fit test, or the Kolmogorov-Smirnov test.

To determine the safety stock level, providing the delivery and demand (consumption) level fluctuate and the length of uncertainty interval is constant, we would suggest using the following method. The fluctuation of the delivery level is expressed by a standard deviation of difference value between inventory on order and actually delivered amount using the formula below [5]

$$x_p = K\sqrt{t_n\left(\sigma_p^2 + \sigma_r^2\right)}, \quad (28)$$

where K – safety factor, t_n – uncertainty interval, σ_p – standard deviation of lead time length t_n, σ_r – standard deviation of difference value r_{xi} between inventory on order x^* and actually delivered amount x_i according to (37).

The safety factor K is a determined quantile of a distribution function of a standardized normal distribution. The value of a safety factor K can be found in charts commonly present in the literature about statistics or by statistic software.

The fluctuation of demand p and uncertainty interval t_n, or lead time is usually measured by sample standard deviations σ_p *a* σ_{tn}, where n is the number of observations;

$$\sigma_p = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}\left(p_i - \overline{p}\right)^2}, \quad (29)$$

$$\sigma_{tn} = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}\left(t_{ni} - \overline{t_n}\right)^2}, \quad (30)$$

where p_i is the actual demand size.
In case the lead time is too long, it is possible to use an alternative approximation method, since the number of the measured values will be very low. This approximation method is based on the knowledge that the standard deviation of lead time length and thus all uncertainty interval is for different theoretical probability distributions about one-fourth of the range. Then, the following approximate equation applies

$$\sigma_{tn} \approx 0{,}25\left(t_{n_{\max}} - t_{n_{\min}}\right). \quad (31)$$

The fluctuation of delivery size x might be expressed either by the standard deviation σ_x of single deliveries size (35), or the standard deviation σ_r of difference value r_{xi} between inventory on order x^* and actually delivered amount x_i according to (36);

$$\sigma_x = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}\left(x_i - \overline{x}\right)^2}, \quad (32)$$

$$\sigma_r = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}\left(r_{xi} - \overline{r_x}\right)^2}, \quad (33)$$

$$r_{xi} = x_i - x_i^*, \quad (34)$$

where p – demand variability, t_n – the interval of uncertainty or lead time, n – the number of observations.

This method is suitable for determining safety stock for overhead material for which we may derive a consumption level in the following period using the facts of a previous period. In armies there are logistics information systems which enable us to perform an accurate analysis of a material consumption level during a previous period.

3. Conclusion

The aim of the inventory management is to obtain a required service level for a reasonable price. This might be achieved by finding a balance between purchase and storekeeping cost on one side and the cost of providing a required service level by customer request. Generally speaking, if the stock volume is high, the service cost is also high as this is the inventory holding cost. Besides this higher cost also the cost of the supply purchasing is higher. There is a potential of financial loss because of the possibility of investing this sum of money in banking and non-banking products. Conversely, if the amount of inventory on hand is low, inventory holding cost will be also low including supply cost. The disadvantage in this case might be a short supply which can be the cause of secondary cost as a result of looking for a stopgap solution. The cost due to a short supply is often very high especially when the production is stopped. In the army this measure is of no importance from the economical point of view. However, this process might include a technical equipment combat efficiency (dependability) requirement at a determined level.

The best solution is to have low cost of service related to spare parts stocking, and spare parts provided in time. It means that there will be no lack of spare parts because of short supply, or only a segment of spare parts will not be available during a maximally determined period of time. This selection segment is performed according to the ABCD method using EW matrix. The key aspect in the stock management is the ability to deal with the uncertainty of the stock.

The presented work has been prepared with the support of the Ministry of Defence of the Czech Republic, Partial Project for Institutional Development, K-202, Department of Combat and Special Vehicles, University of Defence, Brno.

4. References

[1] TENG, Jinn-Tsair. A simple method to compute economic order quantities. European Journal of Operational Research, 2009, 198.1: 351-353.
[2] GRAVES, Stephen C. A single-item inventory model for a nonstationary demand process. Manufacturing & Service Operations Management, 1999, 1.1: 50-61.
[3] CHEUNG, C. F.; WANG, W. M.; KWOK, S. K. Knowledge-based inventory management in production logistics: a multi-agent approach. Proceedings of the Institution of Mechanical Engineers, Part B: Journal of Engineering Manufacture, 2005, 219.3: 299-307.
[4] ALFARES, Hesham K. Inventory model with stock-level dependent demand rate and variable holding cost. International Journal of Production Economics, 2007, 108.1: 259-265.
[5] GOYAL, S. K.; GIRI, Bibhas Chandra. Recent trends in modeling of deteriorating inventory. European Journal of operational research, 2001, 134.1: 1-16.