PROCEDURE FOR THE SEPARATION SEQUENCE OF VERTICES IN DIFERENT ROUTES

Assist prof Berov T.,
VTU Todor Kableshkov, Sofia
E-mail: tberov@vtu.bg;

Abstract: *The objective of vehicle routing problem (VRP) is to deliver a set of customers with known demands on minimum-cost routes originating and terminating at the same depot. Similar to most GA that a chromosome S is a permutation of n positive integers, such that each integer is corresponding to a customer without trip delimiters. Christian Prins proposed an optimal splitting procedure to get the best solution, respecting to a given chromosome. In this paper, application of this splitting procedure to get the best solution, respecting to a sequence of vertices, produced by the heuristic approaches(or a new chromosome produced by the mutation procedure), is considered.*

KEYWORDS: TRANSPORT, VEHICLE ROUTING, HEURISTICS, GENETIC ALGORITHMS, SPLIT DELIVERY

1. Introduction

In the vast majority of the literature, finding tours visited by commercial vehicles is considered as a Vehicle routing problem. The vehicle routing problem (VRP) consists of designing *m* vehicle routes of least total cost and: each starting and ending at the depot, such that each customer is visited exactly once; the total demand of any route does not exceed the vehicle capacity, and the length of any route does not exceed a preset maximal route length. The basic VRP can be extended by taking into account: time windows of customers' requests, heterogeneity of vehicle fleets, tasks conducted by vehicles, the number of home depots, and operational restrictions faced by vehicles.

A large number of algorithms have been developed to solve the VRP problem. Depending on whether an exact optimal solution or an approximate solution is reached, they can be categorized as the exact algorithms or the heuristic methods.

There are different families of heuristics for the VRP. They can be classified into two main groups: the classical heuristics, developed mostly between 1960 and 1990, and the metaheuristics, developed after this period.

The classical heuristics perform a relatively limited exploration of the search space and generally produce good quality solutions within modest computing times. The typical classical heuristics include the well-known savings algorithm (Clark and Wright, 1964), the sweep algorithm (Gillett and Miller, 1974), the petal algorithms (Balinski and Quandt, 1964; Ryan et al., 1993; Renaud et al., 1996), the cluster-first-route-second algorithms (Fisher and Jaikumar, 1981), and the improvement heuristics (Lin, 1965; Thompson and Psaraftis, 1993).

Compared with classical heuristics, metaheuristics perform a much more thorough search of the solution space, allowing inferior and sometimes infeasible moves, as well as recombinations of solutions to create new ones.

Exact algorithms: Branch-and-bound; Branch and Cut Method etc.

Classical heuristics: Route construction heuristics; Savings algorithm - Clarke and Wright (C&W); Two-Phase Methods; sweep algorithm; Solution Improvement; λ-opt heuristic etc.

Metaheuristics: Simulated Annealing; Deterministic Annealing; Tabu search; Genetic Algorithms; Ant System; Neural Algorithms etc.

Metaheuristics commonly used the initial solutions, typically created with some cheapest insertion heuristic.

In this paper is proposed a method for evaluation of the heuristic approaches (or new sequence, obtained by the procedure of mutation), in order to optimize them.

2. Problem Description

As mentioned above, for usage of sequence of vertices, derived from the use of heuristic approaches, we need of splitting procedure, that give an adequate solution, ie with a cost equal to or better obtained by using the initial Heuristic approach. Based on a literature review, widely used is *Splitting algorithm* [6]. In its application, it was found that the result obtained by the algorithm, was not adequate to that, obtained of the used heuristics, as shown in Table 1. If output routes and their first-last vertex are arranged in a certain sequence, i.e. last vertex of a route and first of the next route are in the same cluster, then the result could be with even greater differences. For example, about the problem R101 [7] was obtained:

- „Clarke & Wright", classical: 10 routes, total mileage - 892,464;
- „Clarke & Wright" + opt.: 17 routes, total mileage – 1 361,639.

Table 1: *Problems and result from used models for optimization*

prob.	method	R,*nr*	cost	method	R,*nr*	cost
C1	C&W	10	892,464	+ opt.	11	1 070,284
C1	Sweep	10	1 096,739	-	22	1 860,281
R1	C&W	8	888,266	-	10	1 094,554
R1	Sweep	8	1 029,858	-	10	1 150,004
C2	C&W	3	632,432	-	3	740,894
C2	Sweep	3	800,268	-	5	950,471
R2	C&W	1	523,204	-	1	523,204
R2	Sweep	1	620,492	-	1	620,492
RC1	C&W	6	676,175	-	6	834,543
RC1	Sweep	5	556,480	-	18	1 738,847
RC2	C&W	2	693,330	-	2	724,657
RC2	Sweep	2	809,187	-	2	809,187

To improve the result, we will investigate the possibility of using two main parameters - the distance of each vertex from the Depot and the distance from the previous one, according to the proposed order. They are applied to modernize the classic algorithm of Clarke & Wright [1]. Using these two parameters is shown in (3).

$$S_{ij}=C_{i0}+C_{0j}-C_{ij} \quad (1)$$

here S_{ij} is the corresponding savings and C_{ij} – corresponding cost.

Several improvements to the C&W algorithm have been proposed, to lead to better results overall.. Gaskell and Yellow (1967) have suggested using a positive parameter λ (the *route shape parameter*), through which taken of the relative importance of direct arc between two customers in the calculation of "savings". Paessens (1988) introduce in the model weight μ , for " asymmetric" solving (the distance from the depot for each of the pairs considered customers). The formula for the resulting savings is as follows:

$$S_{ij}=C_{i0}+C_{0j}-\alpha C_{ij}+\mu(C_{i0}-C_{0j}) \quad (2)$$

Here λ is a parameter that controls the relative significance of direct arc between two customers and μ is the asymmetry between two customers with respect to their distances to the depot.

To take into account the use of the vehicle capacity, Altinel and Öncan introduce another parameter, aiming at increase the loading of vehicles. The principle is – „*larger combined route is better*".

(3) $$S_{ij}=C_{i0}+C_{0j}-\alpha C_{ij}+\mu\left(C_{i0}-C_{0j}\right)+\gamma\left(d_i-d_j\right)/\bar{d}$$

here $\overline{d}$ is the average demand of all customers, d_i is the demand of customer i.

Since the chromosome may be broken into several diferent routes, Prins [6] proposed an optimal splitting procedure, which can find the optimum split, ie routes, by minimizing the total cost. The main idea can be described as follows. Let S = (1,2,3..,n) be a given chromosome. Based on the auxiliary graph H = (V',E'), where vertices V'={0,1,2..,n}, and arc $E_{ij} \subset E'$:

(4) $$E_{ij}=c_{0,i}+\sum_{k=i+1}^{j-1}\left(t_k+c_{k,k+1}\right)+t_j+c_{j0}\le L;\ \sum_{k=i+1}^{j} q_k\le Q$$

here t_i is the service time at customer, q_k - the demand for customer.

Then E_{ij} is the total travel cost (time) for the route (i+1, i+2,.., j). An optimal split for S corresponds to shortest path P from vertex 0 to vertex n in H.

```
V0 = 0;
for (i = 1;i<n; i++){Vi = +∞; Pj = 0;}
for (i = 1;i<n; i++){
    cost = 0; load = 0; j = i;
    repeat
        load = load + qsj;
        if ( i = = j)   cost = C0,sj +dsj + Csj;0;
        else {   cost = cost – Csj-1;0 + Csj-1,sj + dsj + Csj,0;
              cost' = α. Csj-1,sj + µ.(abs) (Csj-1;0 - Csj,0);
               cost = cost + cost'; }
    if ((cost ≤ L) && (load < Q)) {
        if  ((Vi-1 + cost) < Vj ) {
                                   Vj = Vi-1 + cost;
                                   Pj = i – 1;   }
        j := j + 1;
    }
    until ((j > n) || (cost > L) || (load > Q))
}
```

Figure 1 *A modified algorithm by using of the coefficients*

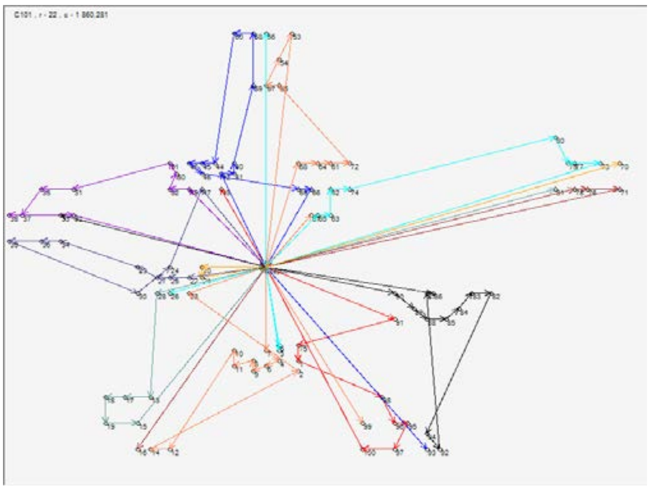

Figure 2 *Routes for C1-Sweep, produced by splitting procedure*

In [6] by example the method was demonstrated. The auxiliary graph helps us understand the idea how to split a given chromosome S into optimal routes. But, we do not have to construct such graph H. It can be done by a labeling algorithm and a splitting procedure [2]. Let S = (1, 2, .., n) be a given chromosome. Two labels V_j и P_j for each vertex j in S are computed. V_j is the cost of the shortest path from node 0 to node j in H, and P_j is the predecessor of j on this path. The minimal cost is given at the end by Vn. For any given i, the increment of j stops when L or Q are exceeded. The labeling algorithm is shown in Figure 1.

3. Application of the proposed algorithm

The influence of the coefficients α and μ was studied, for variant C1 Sweep with 22 routes, Figure2.

In Table 2 gives the parameters for the routes of the viewed variant.

Table 2: *Routes for C1-Sweep, produced by splitting procedure*

№	sequence	$Q_{nc.}$	cost
1	0, 90, 89, 88, 85, 84, 83, 82, 94, 92, 87, 86, 0	190	130,061
2	0, 91, 75, 1, 98, 96, 95, 97, 100, 0	150	111,786
3	0, 93, 0	40	86,023
4	0, 99, 0	10	67,082
5	0, 5, 3, 0	20	32,257
6	0, 7, 4, 6, 9, 8, 11, 10, 12, 14, 2, 23, 0	170	119,064
7	0, 26, 0	10	31,623
8	0, 28, 13, 17, 18, 19, 15, 0	140	90,800
9	0, 16, 0	40	80,623
10	0, 20, 21, 0	30	22,198
11	0, 22, 25, 27, 29, 34, 36, 39, 30, 24, 47, 0	160	105,041
12	0, 49, 52, 50, 51, 31, 35, 37, 38, 32, 0	150	97,873
13	0, 33, 0	40	67,052
14	0, 43, 0	10	33,106
15	0, 42, 41, 40, 59, 58, 60, 44, 45, 48, 46, 69, 66, 0	180	117,468
16	0, 68, 64, 61, 72, 55, 57, 54, 53, 0	150	105,164
17	0, 56, 0	30	90,000
18	0, 67, 0	10	24,413
19	0, 65, 63, 62, 74, 80, 79, 77, 73, 0	170	122,431
20	0, 81, 0	30	94,868
21	0, 78, 76, 71, 0	50	114,298
22	0, 70, 0	30	117,046

The results obtained are shown in Tables 3,4,5,6.

Table 3: *Results for the influence of the coefficient* ***α***

α	routes	cost	α	routes	cost
0	22	1 860,281	2	19	1 450,715
0,2	22	1 866,047	2,5	20	1 439,570
0,4	20	1 735,151	3	21	1 459,526
0,6	17	1 571,181	4	22	1 498,915
0,8	15	1 473,184	10	35	2 039,400
1	14	1 435,887	20	75	4 013,692
1,2	15	1 393,637	30	91	5 111,453
1,6	16	1 323,340	105	100	5 770,962

Table 4: *Results for the influence of the coefficient* ***μ***

μ	routes	cost	μ	routes	cost
-1	24	1 891,373	4	19	1 433,824
-0,6	25	1 923,156	5	20	1 454,848
-0.4	25	1 901,894	10	31	1 875,220
0	22	1 860,281	15	44	2 533,716
0,2	20	1 808,386	30	70	3 950,366
0,6	16	1 548,751	50	84	4 682,807
1	15	1 480,879	100	94	5 300,032
1,5	15	1 448,808	200	95	5 402,110
3	17	1 401,447	300	97	5 529,406

Table 5: *Results for the influence of the coefficients* ***α*** and ***μ***

μ	α	routes	cost
0,15	1	14	1 349,104
0,5	1	15	1 382,475
0,8	1	17	1 462,267

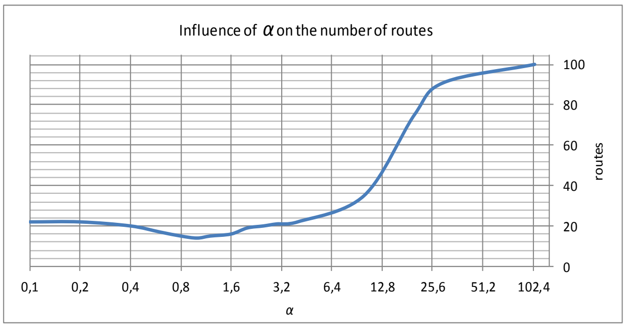

***Figure* 3** *Influence of α on the number of routes*

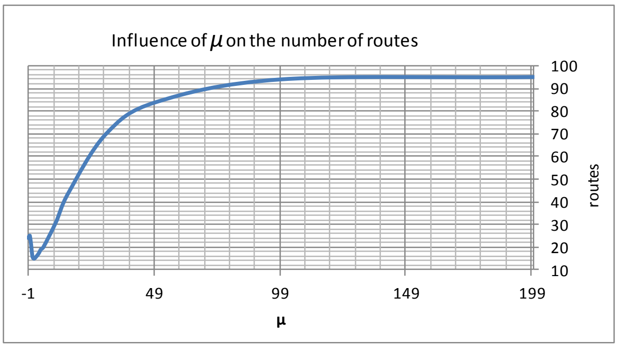

***Figure* 4** *Influence of μ on the number of routes*

4. Results

The influence at issue parameters, independently apart, on the number of received routes are illustrated in Figure 3 and 4. In Table 5 is shown the results obtained when combining them, and in table 6 are the routes and cost, corresponding for the best combination. When comparing the obtained results with those of Table 7, we see the difference in the number of routes - 4 and about 23% in cost.

The results obtained at researches done with other test examples, shown in Table 8, are closer to the input variants.

Table 6: *Routes for C1-Sweep, produced by splitting procedure and* ***α*** = 1, ***μ*** = 0,15

№	sequence	$Q_{nc.}$	cost
1	0, 90, 89, 88, 85, 84, 83, 82, 94, 92, 0	160	116,723
2	0, 87, 86, 91, 75, 1, 98, 96, 95, 97, 100, 0	180	123,730
3	0, 93, 99, 0	50	87,733
4	0, 5, 3, 7, 4, 6, 9, 8, 11, 10, 12, 14, 0	150	97,423
5	0, 2, 23, 26, 28, 0	70	66,007
6	0, 13, 17, 18, 19, 15, 16, 0	160	93,117
7	0, 20, 21, 22, 25, 27, 29, 34, 36, 39, 0	160	83,916
8	0, 30, 24, 47, 49, 52,50,51,31,35,37, 38, 32,0	180	124,161
9	0, 33, 0	40	67,052
10	0, 43, 42, 41, 40, 59, 58, 60, 44, 45, 48, 46, 0	170	104,449
11	0, 69, 66, 68, 64, 61, 72, 55, 57, 54, 53, 0	170	107,745
12	0, 56, 0	30	90,000
13	0, 67, 65, 63, 62, 74, 0	140	43,056
14	0, 80, 79, 77, 73, 81, 78, 76, 71, 70, 0	150	143,989
Total			1349,101

Table 7: *Optimal routes for C1Sweep*

№	sequence	$Q_{nc.}$	cost
1	0, 90, 89, 88,85,84,83 82,94, 92, 87,86, 91, 0	200	133,756
2	0, 75, 1, 98, 96, 95, 97, 100, 93, 99, 0	190	106,068
3	0, 5, 3, 7, 4, 6, 9, 8, 11, 10, 12, 14, 2, 0	180	106,141
4	0, 23, 26, 28, 13, 17, 18, 19, 15, 16, 0	200	100,336
5	0, 20, 21, 22, 25, 27, 29, 34, 36, 39, 30, 24, 0	180	88,037
6	0, 47, 49, 52, 50, 51, 31, 35, 37, 38, 32, 33, 0	200	102,594
7	0, 43, 42, 41, 40, 59, 58, 60, 44, 45, 48, 46, 0	170	104,449
8	0, 69, 66, 68, 64, 61, 72, 55, 57, 54, 53, 56, 0	200	111,568
9	0, 67, 65, 63, 62, 74, 80, 79, 77, 73, 0	180	122,831
10	0, 81, 78, 76, 71, 70, 0	110	120,957
Total			1096,739

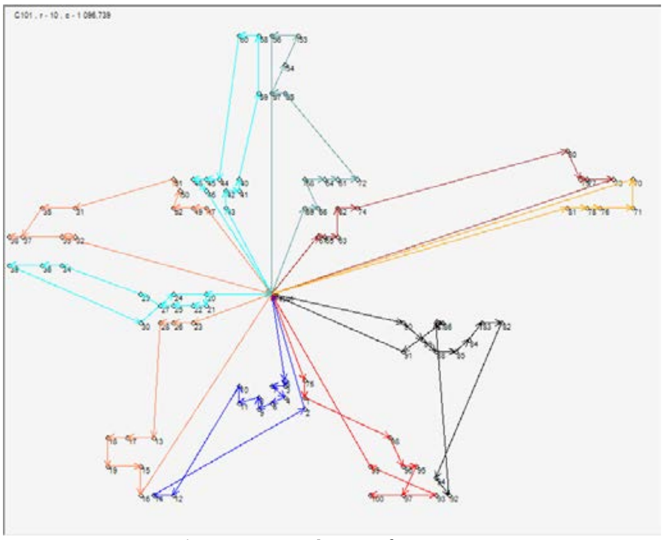

***Figure* 5** *Optimal routes for C1Sweep*

Table 8: *Problems and results of applications optimization model*

Пробл.	Method	rout.	cost	μ	α
C1	C&W	10	922,619	0,3	0,2
C1	Sweep	10	1349,101	0,15	1
R1	C&W	9	949,878	0,2	0,4
R1	Sweep	9	1 088,836	0,2	0,4
C2	C&W	3	632,432	0,2	0,4
C2	Sweep	4	869,468	0,2	0,4
RC1	Sweep	7	830,142	0,2	0,7

From these results is seen that with the introduced modifications, we do not achieve the desired final results.

We can get the desired final results, using these coefficients with higher values, and achieve variant with more routes. Through an additional procedure for their optimization (applied to the "label" array P_j ,Figure 1), was prepared desired result.

5. Conclusions

A large number of algorithms have been developed for obtaining the routes of commercial vehicles, ie to solve the VRP problem. При мета-евристиките широко се използва евристика за получаване на изходен вариант. Particular attention must be paid to then local search method.

The proposed modification, by examining the two parameters to the decoupling algorithm generally has a positive effect, and through subsequent optimization, we obtain the result, with the parameters of the heuristics.

The proposed modification, using two parameters examined, to the splitting algorithm generally has a positive effect, and through subsequent optimization, we obtain the result, with the parameters of the initial heuristics.

The method is used for splitting a sequence of vertices resulting from the procedures for the drawing-up of new sequences by genetic algorithms, and also as a procedure for getting the routes after the use of the algorithm for the Traveling salesman problem.

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Acknowledgements: *The paper is published with the support of the project BG051PO001-3.3.06-0043 "Increasing, Improving and Extending the Scientific Potential of the University in Transport by Support to Development of PhD Students, Postdocs, Trainees and Young Researchers in the Field of Transport, Power Engineering and ICT in Transport" within the Human Resources Development Operational Programme co-funded by the European Social Fund of the European Union.*