EQUIVALENT TRACK STIFFNESS DETERMINATION

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Abstract: *In the paper the so called equivalent stiffness of track was defined as a parameter describing stiffness of track in vertical direction. This stiffness is one of the important quantities deciding of a dynamical behaviour of railway vehicle. To determine the above-mentioned stiffness the field experiment was carried out using the real track and the car being in use at present. Vertical deflections of rails were measured under the quasi – static load coming from the axles (wheels) of the car moving with velocity close to zero.*

Keywords: TRACKS STIFFNESS VARIATIONS, MEASUREMENTS, TRACK SETTLEMENT

1. Introduction

Generally dynamic behavior of railway track, although its construction is not very complex, belongs to the most complicated problems of railway engineering. Railway track is a place that is a source of excitations in railway vehicle – track system and more over its mechanical properties can change themselves in time of operation. That fact is due to interaction of track components, where several nonlinearities are observed [1] as well as vehicle track interaction and increased vehicle speed. Therefore, existing track models are rather complicated and the extensive survey of such models was given in paper [2]. The progress in track dynamics now is concentrated on problems of its long time behavior especially on problems connected with track settlement for example [3] and forming of track irregularities [4]. Future trends in tracks dynamics researches were introduced in [5]. There has been paid attention to the problems of track ballast pulverization and ballast stiffness changes along the track. As mentioned previously due to complexity, such models of railway track cause computational difficulties. Therefore, it was necessary to build simple models that can be incorporated into existing commercial software packages studying the dynamics of rail vehicle – track system. Besides the simplifications, such simple models would be useful to engineers, which are more involved in studying railway vehicle dynamics then in civil engineering. The development of such track models primarily consists in determining the replacement values of the equivalent mass, damping and stiffness along the railway track.

This paper is a contribution to those problems and presents a method of estimation of the track vertical stiffness changes based on the measured track vertical deflections during quasistatic railway vehicle motion along the track. Measurements of deflections were performed on the selected railway track section for applied static axle load of the wagon. Then the measured deflections were used to estimate track vertical stiffness variations. The courses of these variations were considered on the basis of two series of experiments executed on the same track section at ten months long interval.

The determination of track vertical stiffness and its variation along the track are relatively difficult and expensive. There are not many available references on the experimental methods and the results obtained. Moreover, published results of track vertical stiffness values are frequently divergent, because they refer to different design and state of track maintenance, e.g. [6]. These facts were the motivation to undertake experimental research, which are presented in this paper.

2. Models of track-railway vehicle system

In the paper two models of track railway vehicle system are presented. The difference between them is in the part representing track and its vertical stiffness.

Model I is presented on fig. 1.

Rail track is modeled as a continuous infinite Bernoulli-Euler beam resting on elastic foundations characterized by vertical stiffness coefficient k [N/m^2] on which the model moves with the wheelset acting vertically on the track.

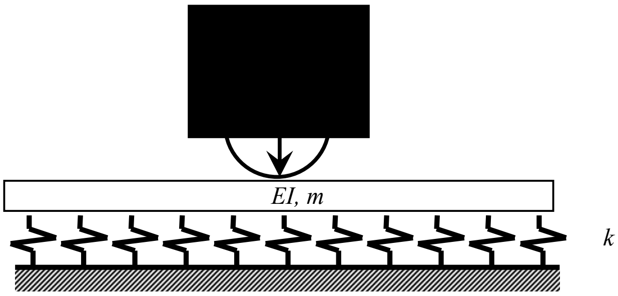

Fig. 1 *Model I, Q[N] – vertical force acting from the wagon wheelset, k[N/m^2] – equivalent track stiffness, m[kg/m] - distributed track mass.*

Assuming that loads from other wheelset do not influence considered wheelset, its defelection is defined by the following equation.

$$EI\frac{\partial^4 y}{\partial x^4}+m\frac{\partial^2 y}{\partial t^2}+ky=q(x)$$

where:

EI - bending stiffness of beam,

k - vertical stiffness of elastic foundations,

q(x) - continuous load,

m - track mass.

In the case considered q=0, and Q is included in boundary conditions. Boundary conditions are assumed as zeros for displacement, angle and torques at the beam ends. Neglecting the efect of mass (it is allowed in static case of loading) the solution of above equation can be expressed in a form convenient to interpret:

$$y(x)=\frac{Q}{2kL}u(x)$$

where:

$$L=\sqrt[4]{\frac{4EI}{k}}$$

L - is a characteristic length, depending on the bending stiffness *EI* of rails and the coefficient of the foundation (ballast) *k*.

$$u(x)=-e^{-x/L}[\cos\frac{x}{L}+\sin\frac{x}{L}] \quad \text{for } x\geq 0$$

$$u(x)=-e^{x/L}[\cos(-\frac{x}{L})+\sin(-\frac{x}{L})] \quad \text{for } x\leq 0$$

u (x) - normalized line of track centre line deflection (relative deflection).

Knowing deflections of rails we can calculate stiffness k from the above equation using finite element method model of infinitive beam.

Model II is presented on fig. 2.

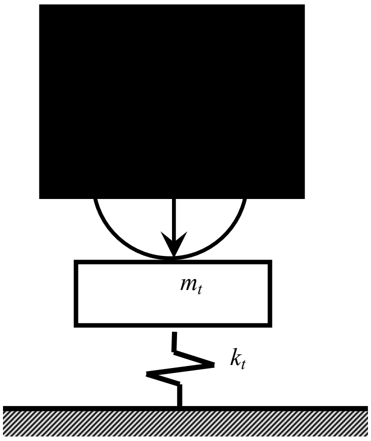

Fig. 2 *Model II, Q[N] - vertical force acting from the wagon wheelset, m_t[kg] - equivalent mass of rail track, k_t - equivalence track stiffness coefficient.*

The track is modelled as a discrete mass m_t supported on a spring characterized by the equivalent stiffness coefficient k_t.

As the model moves both m_t and k_t change. The equivalent stiffness of the track can be obtained when the load Q and deflection of the point of track under the load is known. These data are obtained from a field experiment.

The equivalent mass of the track m_t is not taken into account as small compared to the mass of a wagon. When it is necessary the way to determine it can be found in [1]. It is shown there that $m_t \approx 200$ kg.

3. The field experiment

The field experiment was carried out on the track badly maintained and being used rarely which was necessary as not to close the track for normal traffic. That experiment was carried out in two stages at ten months long interval.

Non-contact method was applied for measuring vertical displacements of rails. Vertical deflections of both rails were recorded simultaneously on the same sleeper. To do this, two fast cameras having the high resolution (up to 0.05 mm) have been applied. They were placed on both sides of the track allowing to measure the deflections of two rails above the same sleeper. On the rails were located markers equipped with the number of sleeper and scale. Those allowed the reading of deflections.

Analysis of measured data consisted of processing of movies during subsequent passes of the train. To do this, the selected sections of each video frames were recorded one after the other as a bitmap. In this way, the rail deflections courses were obtained for the sleeper vs. time. The interval between film frames should be understood as the time. This interval was 0.02 seconds in the case. Then, based on recorded changes in the positions of markers and fixed transformation coefficient η [mm/pixel], the bitmap sets were transformed to obtain the actual deflection of the rails. The results were recorded in the Excel worksheets.

During the experiment the two axle wagon with a long (9 meters) distance between axles moved not faster the 5 km/h over the track section whose deflection was measured. the slow velocity let us avoid the dynamical effects. The long distance between axles was necessary to avoid the influence of the load of one axle on a deflection of another.

Vertical deflections of rails, caused by quasistatic train passage, were recorded for subsequent wheelsets of test wagon above each measurement sleeper. The wagon was suitably loaded to reach axle loads: 100, 150 and 200 kN. In fact, on the experimental track section of 66.6 m length 111 sleepers were located (56 measured sleepers).

Slightly different concept of experiment was presented by Froehling [3]. The experiment was designed to estimate the vertical stiffness of the ballast and the rate of track settlement. Besides that, there was used a different measurement technique.

The deflection was measured eaxctly at the moment when the load was over the point of measurement, separately for left and right wheel. The example of the results obtained for different axle loads for second stage of experiment is shown on fig. 3.

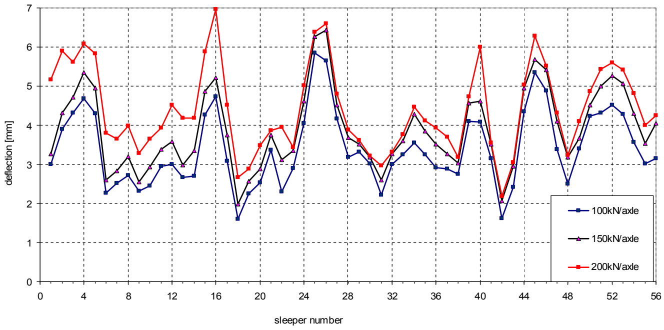

Fig. 3 *Changes of the vertical deflections.*

It can be noticed that the deflection changes strongly along the track what can be the result of poor technical maintenance of the track.

Displacements of the left and right rails on the same sleeper generally differ from each other, while the ranges of these changes for each axle load are comparable. The differences of displacements for the left and right rails may be caused by no symmetric mass distribution of the test car, the heterogeneity of ballast (in terms of its vertical stiffness) or different mutual adjacency of the superstructure to the ballast on the left and right side.

The statistical parameters of the vertical deflection of the track are shown at table 1.

Table 1: *Statistical parameters of track deflection.*

Statistical parameters [mm]	First axle	Second axle	Axle load [kN]
μ	3,29	3,27	100
σ	1,20	1,21	
μ	3,82	3,83	150
σ	1,38	1,40	
μ	4,31	4,22	200
σ	0,89	0,93	

Basing on fig. 3 and table 1 we can easily obtained the equivalent stiffness coefficients of the track for the Models I and II. The equivalent stiffness coefficient for Model I is shown on fig. 4 and its statistical parameters in table 2.

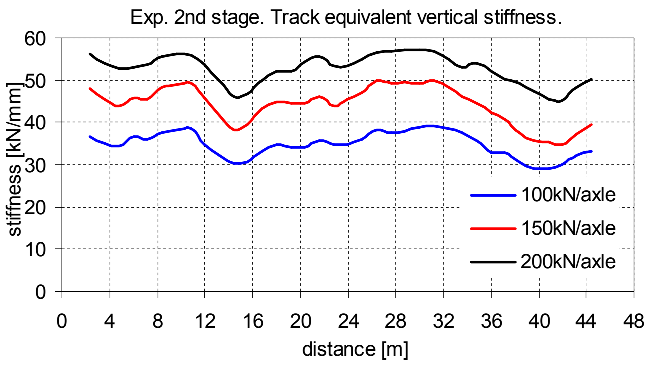

Fig. 4 *Changes of equivalent vertical track stiffness determined on the basis of track deflections measurements.*

Table 2: *Mean values and standard deviations of equivalent vertical track stiffness.*

Statistical parameters [kN/mm]	100 kN/axle	150 kN/axle	200 kN/axle
Mean value μ	34,76	44,16	52,61
Std. deviation σ	2,43	6,15	4,19

From table 1 we can easily calculate the equivalent stiffness coefficient k_t for the Model II. The average value of k_t for the axle load 100 kN is 30.4 kN/m, for load 150 kN 39.0 kN/m and for load 200 kN 46.4 kN/m.

4. Conclusions

The experiment mentioned above enable us to calculate the stiffness coefficient for Model I and II. Model II seems to be more useful when model of vehicle is discrete.

Using the average coefficient of stiffness is not proper when the maintenance of track is poor because there is a big difference between minimum and maximum of this coefficient.

The experiment should be repeated for well maintained track. It can be expected that the above mentioned difference will be relatively small and using the average coefficient will be acceptable.

The experiment proved that the value of the equivalent coefficient of stiffness in one point of a track changes depending on load. It is apparent from the courses of the stiffness change that characteristics of the track displacements depend nonlinearly on this force As the deflection of the well maintained track is small (smaller then when the maintenance is poor) the assumption that k_t and k as constant can be proper.

Proposed method allows to estimate track equivalent vertical stiffness and its changes along the track.

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