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MODELING OF PROCESSES IN TRANSPORT SYSTEMS BY FRACTAL METHODS

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Abstract: *In the paper is proposed a model for studying of processes in a transport system by fractal methods. In view of the NDS general properties and examined categories: order, chaos and self-organization, the study is based on the synergistic approach to developing systems. On the basis of this has been performed research in the environment Matchcad with different values of system's parameters.*

Keywords: TRANSPORT SYSTEMS, FRACTALS, DETERMENISTIC CHAOS, SELF-SIMILARITY, SCALE INVARIANCE

1. Introduction

Transport systems are complex technical systems consisting of a large number of interrelated elements: traffic participants, set of vehicles, a significant number of transportation process operations. They are characterized by interaction of heterogeneous traffic of passenger flows and a certain number of vehicles involved in the movement at the current time. This reflects a combination of deterministic and stochastic factors in the operation of transport systems.

Transport systems are nonlinear and deterministic, but exhibit complex chaotic behavior. They have a large number of possible states (stable and unstable), the transition between which is possible at any time. All of these features provide a basis to identify a transport system with a nonlinear dynamical system (NDS) [1].

For study of processes in transport systems general methods are used: physical experiments [2], visual diagnostics of the process [2], numerical methods [3], computer simulation [3]. In the scientific literature there are examples of application of the theory of nonlinear dynamics for prediction of motorised traffic flows [4,5,6]. In [7] is analyzed traffic flow data and characterized it as chaotic [8] showed that chaotic behavior in traffic can due to the delays in human reaction. Here a synergistic approach is manifested, designed to describe phenomena in the world of nonlinear systems, actively interacting with external environment. Their behavior has considerable dynamics and is described by similar mathematical models.

In the paper is proposed a model of examination of processes in a transport system by fractal methods. The existence of a fractal nature was investigated using the identification properties of chaotic processes [9,10].

2. The most important properties of chaotic processes

The first signs of the presence of chaotic properties of the processes are identified by observing time dependence. If during visual inspection signs of order or periodicity are not detected, it can be assumed that the process is chaotic.

Another method for visual diagnosis is associated with receipt and investigation of phase portraits. To characterize the attractor it is advisable to define its dimension, since non-integer fractal dimension indicates the presence of a fractal structure.

The chaotic processes are strongly dependent on initial conditions and susceptible to external influences. From the theory of nonlinear dynamics [1] it is known, that a small change in chaotic system's parameters may lead to changes in the type of solutions.

Methods that give a numerical estimate of chaotic state of the system, definition of autocorrelation function and calculation of capacity power spectrum of chaotic oscillations. For a chaotic process, the autocorrelation function decays exponentially. The spectrum obeys the law form:

$$S_x(f) = Bf^{\beta} \quad (1)$$

where f is the frequency and β is a constant.

Undoubtedly, however, the most important characteristics of the chaotic process are Lyapunov exponents and their spectrum [1]. A necessary and sufficient condition for chaotic state of the system remains positivity property of senior Lyapunov exponent [1].

Similar to the Lyapunov Exponents, a well-established parameter that is commonly used for testing for the chaos in systems is the Hurst exponent - *H* [1]. The Hurst Exponent, *H,* is a measure of the degree to which a given time series can be statistically expressed as a random walk (i.e., Brownian motion).

Another property of chaotic processes is the self-similarity [1]. It is the property we associate with fractals — the object appears the same regardless of the scale at which it is viewed. As the Hurst parameter *H* increases the degree of self-similarity is increasing ($0.5 < H < 1$).

3. Modeling of a transport system

Condition of transport system can be defined as a set of certain values

x_i , $i = 1,...,n$.

The meaning and the number of variables x_i may be different. They are directly related to observed quantitative characteristics of the transport system.

As an example, in the simplest case when the dimension of the phase space is n=3:

-x_1 is a number of vehicles involved in the transport process at a given time;

-x_2 is a number of passengers;

-x_3 is a number of seats in vehicles.

The state of the transport system is described by system of nonlinear ordinary differential equations such as:

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, t, \mu) \quad , \quad (2)$$

where $\boldsymbol{x}=(x_1, x_2, x_3)^T$;

t is time;

μ is a set of parameters;

$\boldsymbol{f}= (f_1, f_2, f_3)^T$ are nonlinear functions.

The mathematical model (2) reflects the dynamic and active relations in the transport system.

The right elements of the equations (2) may be different depending on the kind of the functional dependence $\boldsymbol{f}$ and parameter values. The parameters of this kind include, for example, the intensity of the change in variables, maximum capacity of vehicles, etc. With such a basis the mathematical model (2) will reflect basis cause effect relationships of the transport system functioning and take into account its multi-element character.

On the basis of causes of changes in variables and their interrelationships, it is assumed that:

- change in the number of participating vehicles depends on presence of seats and a certain number of passengers in them;

-change in the number of passengers served depends on the number of available transport vehicles and the number of seats in them;

-change in the number of free seats is in proportion to the total number of vehicles, the number of seats in each of them and the number of passengers.

Then the model of transport system can be represented as:

$$\begin{cases} \dot{x}_1 = cx_3 - ex_2 \\ \dot{x}_2 = ax_1 + bx_2 \\ \dot{x}_3 = x_1 - dx_3 - x_2 \end{cases} \quad . \qquad (3)$$

Parameters *a*, *b*, *c*, *d*, *e* in (3) have in relative terms the following values:

-*a* is coefficient taking into account the need to increase the vehicles in operation;

-*b* expresses the possibility of overload;

-*c* takes into account the percentage of occupied seats in vehicles;

-*d* reflects the marginal occupancy of seats;

-*e* expresses the intensity of passenger flow.

The system (3) is solved numerically in the software environment Mathcad for different values of parameters and initial conditions. Timing diagram, phase portrait and trajectories are obtained in Fig. 1, Fig.2 and Fig.3.

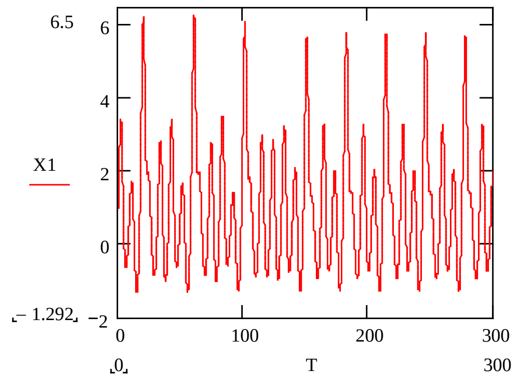


Fig. 1 Timing diagram

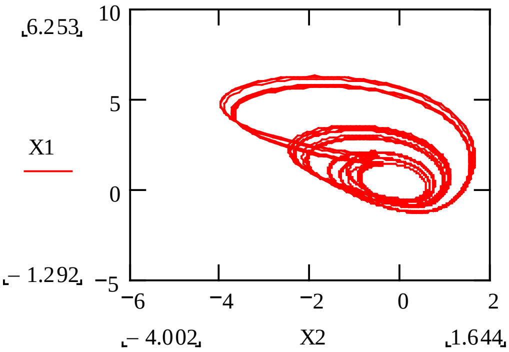
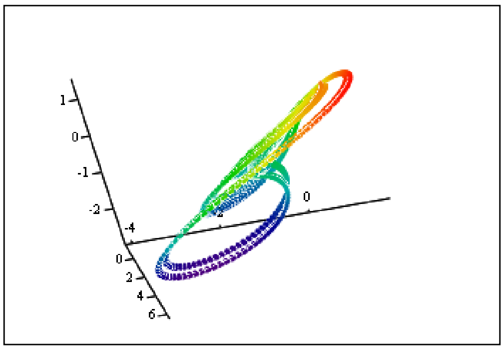


Fig. 2 Phase portrait

As can be seen from Fig.2 and Fig.3, the phase space of the system for certain values of the parameters is a strange attractor, which confirms the chaotic behaviour. In the time dependence from Fig.1 lacks periodicity of phase variables.

To verify the sensitivity of the solution to initial values time-base dependencies of system solutions with different initial values (Fig.4).



(X1, X2, X3)

Fig. 3 Phase trajectories

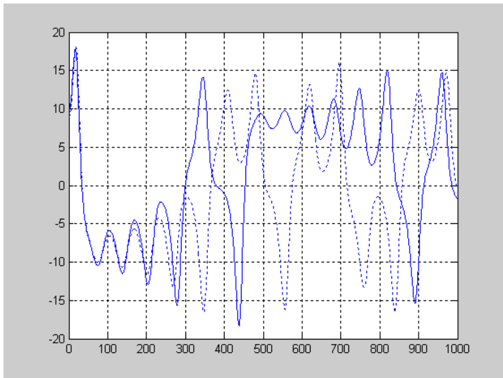


Fig.4 Time-base dependencies of system solution with different initial values

The resulting graph facilitates a preliminary conclusion about the exponential instability of trajectories, that is, about existence of a positive indicator in the spectrum of Lyapunov exponent.

The autocorrelation function of process (Fig.5) decays exponentially.

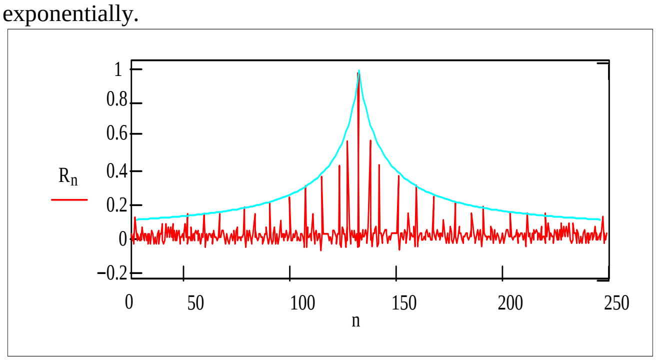


Fig.5 Autocorrelation function

4. Conclusion

A model by means of nonlinear dynamics is proposed, which from the standpoint of synergistic understanding in phenomena in the world of nonlinear systems is applicable to the study of transport processes. Using it, it is possible to find an answer to the question regarding identification of system parameters' values that cause chaotic behaviour. This method allows to control the dynamics of transport systems in order to achieve the desired mode of operation.

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