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HYDRO-MAGNETIC FLOW OF A VISCOUS FLUID WITH HEAT TRANSFER PHENOMENA BETWEEN TWO VERTICAL PLATES

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Abstract: *The paper aims to study the combined effects of uniform magnetic field and inertia on a three-dimensional flow of a viscous fluid in a vertical channel. It is assumed that the fluid is injected into the channel through one side of the channel. The flow and heat transfer phenomena have been characterized by non-dimensional parameters R (cross-flow Reynolds number), M (magnetic parameter), Pe (Peclet number), and α (heat generation parameter). The basic equations governing the flow and heat transfer are reduced to set of ordinary differential equations using the appropriate transformations for velocity components and temperature. These equations have been solved approximately subject to be relevant boundary conditions by employing a numerical technique. The effects of the above-mentioned parameters on the velocity and temperature distributions have been examined carefully.*

Keywords: VISCOUS FLUID, HYDRO-MAGNETIC FLOW, HEAT TRANSFER.

1. Introduction

The problem of flow through a channel with porous walls has attracted the attention of mathematicians and engineers in view of its multiple applications. Examples of these applications are the boundary layer control, transpiration cooling, gaseous diffusion, preventing corrosion, reactions addition and drag reduction. Berman [1] was the first to study a two dimensional, incompressible, steady, laminar suction flow of a Newtonian fluid in a parallel-walled porous channel. He found a series solution for the case of a very low cross-flow Reynolds number. After his pioneering work, this problem has been studied by many researchers considering various variations in the problem, e.g., Choi et al. [2] and references cited therein. Wang and Skalak [3] considered the three-dimensional problem of fluid injection through one side of a long vertical channel for the Newtonian fluid. They obtained an approximate solution and integrated numerically for comparison. They reported that the perturbation series method is fairly accurate only for low cross-flow Reynolds number. Huang [4] investigated Wang and Skalak's problem by a quasi-linearization technique which relaxes the limitation on the magnitude of the cross-flow Reynolds number. The same flow problem was solved for large cross-flow Reynolds number by Ascher [5] using a spline collocation method. Sharma and Chaudhary [6] reconsider the above-mentioned problem by introducing a second order fluid. They obtained the second order perturbation solution. Baris [7] continued the last mentioned research by substituting thermodynamically compatible fluid of second grade instead of second order fluid. In his subsequent study [8], he extended the analysis of the same problem to a Walter's B elastico-viscous fluid. Recently, Joneidi et al. [9] have found an approximate analytical solution using Homotopy Analysis Method (HAM) for the problem discussed in [8]. The aim of this work is to generalize Wang and Skalak's problem in two directions. The first generalization concern with the flow consideration in the presence of a constant magnetic field and second to consider the effect of heat generation on the temperature distribution.

2. Governing Equations

Figure 1 represents the flow model and coordinate system in a vertical channel. An incompressible fluid is injected through a vertical plate at $y = d$ with uniform velocity V_0 . The fluid strikes another vertical plate $at = 0$. It flows out through the opening of the plates, due to the action of gravity along the z-axis. An external uniform magnetic field B_0 is applied in the y-direction. Since the gap d is small, we may assume that both planes extend to infinity. Due to this assumption the edge effects are ignored and the isobars are parallel to z-axis.

Let u, v, w be the velocity components in the directions of x, y, z respectively. We shall seek a velocity field, compatible with the continuity equation, of the form [3]

$$u = \frac{V_0}{d} x\, f'(\eta)\ ,\ v = -V_0 f(\eta)\ ,\ w = \frac{d^2 g}{\nu} h(\eta) \tag{1}$$

where ν is the kinematic viscosity, g is the gravitational acceleration, $\eta = y/d$ is the similarity variable and the prime denotes the differentiation with respect to η.

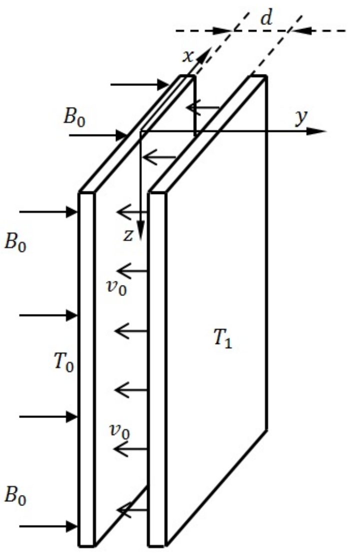

Fig. 1 *Physical model and coordinate system*

The three-dimensional Navier-Stokes and energy equations governing such type of flow are written as:

$$\rho\left[u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right]=-\frac{\partial p}{\partial x}+\mu\left[\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right]+F_x$$

$$\rho\left[u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}\right]=-\frac{\partial p}{\partial y}+\mu\left[\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right]+F_y$$

$$\rho\left[u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}\right]=\ldots$$

$$\ldots-\frac{\partial p}{\partial z}+\mu\left[\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right]+\rho g+F_z \tag{4}$$

$$\rho c_p \left[u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z} \right] = \ldots$$
$$\ldots k\left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right] + Q_0\left(T - T_0\right) \qquad (3)$$

where ρ is the density, p the pressure, μ the dynamic viscosity, T the temperature, c_p the specific heat at constant pressure, k the thermal conductivity, Q_0 the volumetric rate of heat generation, T_0 the temperature of the plate at $y = 0$ and $\mathbf{F} = (F_x, F_y, F_z)$ is the source term due to the imposed magnetic field. The electromagnetic body force per unit volume is can be simplified as:

$$\mathbf{F} = \sigma_0\left(\mathbf{V} \times \mathbf{B}\right) \times \mathbf{B} \qquad (4)$$

where is the electrical conductivity and $\mathbf{B} = (0, B_0, 0)$ is the transverse uniform magnetic field applied to the fluid.

Our investigation is restricted to following assumptions:

(i) All the fluid properties are constant.

(ii) The flow is steady and laminar.

(iii) The plates are electrically non-conducting.

(iv) The magnetic Reynolds number is so small that the induced magnetic field produced by the motion of fluid can be ignored in comparison to the applied one. In addition, the imposed and induced electrical fields are assumed to be negligible.

(v) The effects of viscous dissipation, Ohmic heating and Hall current are not included in the analysis, since they are generally small in the flow region. Also, the radiant heating is neglected.

Substituting Eq.(1) into Eqs.(2)-(4) and eliminating the pressure term from these equations, we arrive at the following equations:

$$f''' + R\left(f\,f'' - f'^2\right) - M^2 f' = C \qquad (5)$$

$$h'' + R\,f\,h' - M^2 h = -1 \qquad (6)$$

where C is an unknown constant, the cross-flow Reynolds number R and magnetic parameter M are defined through, respectively

$$R = \frac{V_0\,\rho\,d}{\mu} \quad , \quad M = B_0\,d\sqrt{\sigma_0/\mu} \qquad (9)$$

The boundary conditions for the velocity field are

$$\eta = 0 \;\; : \;\; f(0) = 0 \;,\; f'(0) = 0 \;,\; h(0) = 0 \;,$$
$$\eta = 1 \;\; : \;\; f(1) = 1 \;,\; f'(1) = 0 \;,\; h(1) = 0 \;. \qquad (7)$$

Next, we introduce a temperature field of the form

$$T = T_0 + \left(T_1 - T_0\right)\theta(\eta) \qquad (11)$$

where T_1 is the temperature of the plate at $y = d$. Using usual non-dimensional procedure and substituting Eq.(1) and (11) into Eq.(5), we get

$$\theta'' + R\,\mathrm{Pr}\,f\,\theta' + \alpha\,\theta = 0 \qquad (12)$$

where $\mathrm{Pr} = \mu c_p/k$ is the Prandtl number and $\alpha = Q_0 d^2/k$ is the heat generation parameter. Note that the product of R and Pr is equivalent to the Peclet number ($\mathrm{Pe} = R * \mathrm{Pr}$) in the context of the transport of heat.

The corresponding boundary conditions are

$$\theta(0) = 0 \quad , \quad \theta(1) = 1 \qquad (13)$$

It is recorded that in the absence of M and α Eqs. (7), (8) and (12) together with the associated boundary conditions (10) and (13) are the same as those obtained by Wang and Skalak [3].

3. Numerical Results

In the presence of a transverse uniform magnetic field and heat generation, the flow and heat transfer problem involving three-dimensional flow between two vertical plates is governed by the similarity equations and boundary conditions given in Eqs. (7), (8), (12) and Eqs. (10), (13). Since the equations have no analytical solutions, they must be solved numerically. Numerical solutions were obtained using the Matlab solver boundary value problem (bvp4c) designed for the solution of two point boundary value problems. The code is based on a collocation formula. An error estimate for the global error of the approximate solution is also provided. Mesh selection and error control are based on the residual of the continuous solution. We set the relative and absolute tolerance equal to 10^{-6} . We refer the reader to the book by Shampine et al. [10] for details about how to solve boundary value problems with bvp4c. As a test of accuracy of the numerical solutions, the results are compared with those known from the literature [3,4,7] and an excellent agreement is found for the case of $M = 0$ and $\alpha = 0$.

We computed the velocity components and temperature field for the problem under discussion by assigning some specific values to the parameters entering into the problem.

The effects of these parameters on the above mentioned fields are presented Figure 2 to 5. Figure 2 depicts the normal velocity component for various values of the magnetic parameter M^2 when R is fixed at 10 and 20. It is clear from this figure that the normal velocity decreases slightly with magnetic parameter. Figure 3 illustrates the effect of magnetic parameter on tangential velocity profiles for the same values of cross-flow Reynolds number.

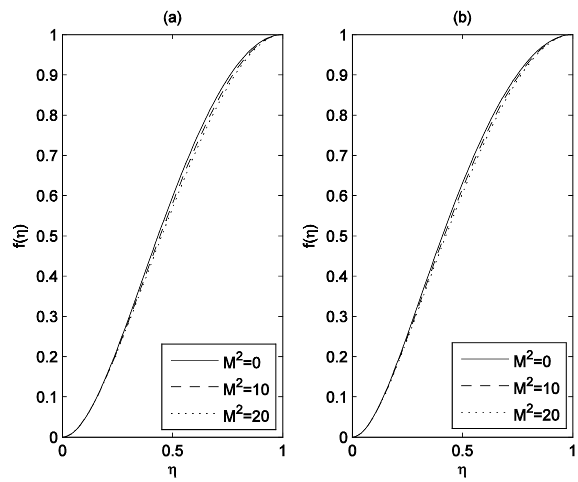

Fig. 2 *(a) Normal velocity profiles for* $= 10$ *, (b) Normal velocity profiles for* $R = 20$

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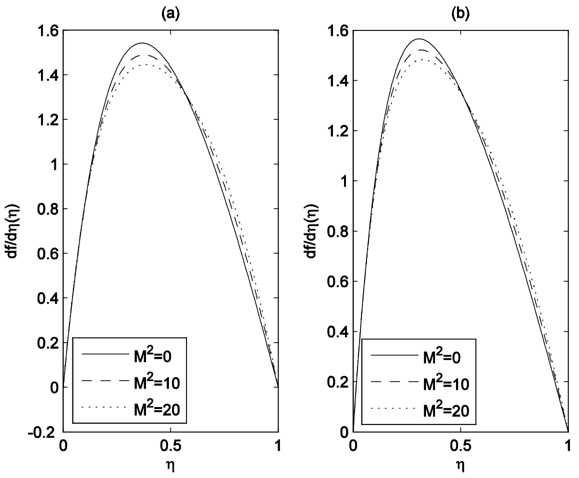

Fig. 3 *(a) Tangential velocity profiles for* $R = 10$ *, (b) Tangential velocity profiles for* $R = 20$

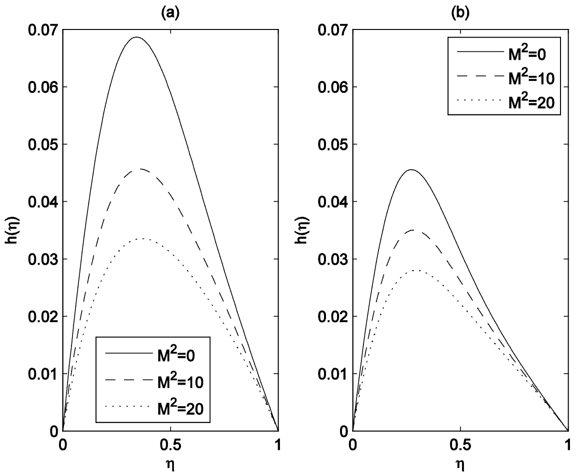

Fig. 4 *(a) Axial velocity profiles for* $R = 10$ *, (b) Axial velocity profiles for* $R = 20$

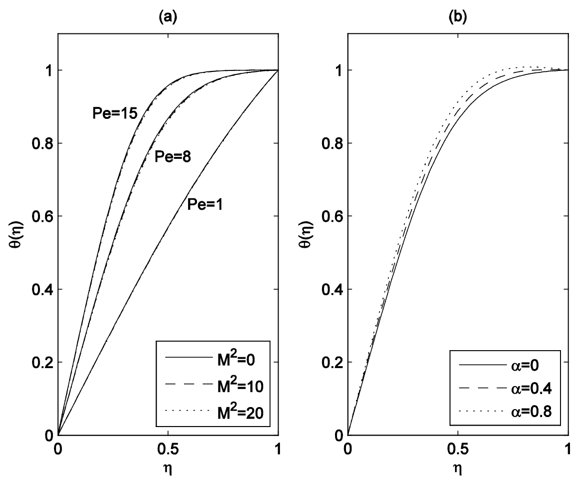

Fig. 5 *(a) Temperature profiles for* $\alpha = 0$*, (b) Temperature profiles for* $Pe = 8$ *and* $M^2 = 1$

4. Conclusion

It is interesting to note that the tangential velocity decreases with an increase in the magnetic parameter, up to the point near the middle of the channel, and thereafter increases with increasing M^2. Again from this figure we observe that with an increase in the value of cross-flow Reynolds number, the point at which maximum tangential velocity occurs moves away from the porous plate. This is because the increment of shear stresses on impermeable surface at $y = 0$. The axial component of velocity is due to the action of gravity along the z-axis. This velocity component is shown in Fig. 4. It is obvious from this figure that the velocity profiles decrease with an increase in the cross-flow Reynolds number. Also increasing the magnetic parameter decreases the axial velocity further. This is expected since the application of a transverse magnetic field normal to the flow direction has a tendency to create a drag-like Lorentz force. This force has a decreasing effect on the axial velocity. To investigate the effect of the Peclet number and the magnetic parameter on the temperature, we have presented the temperature profiles in Fig. 5(a) in the absence of heat generation. Due to the convection effects, increasing of Peclet number leads to intensify the temperature distribution in the channel. Again from Fig. 5(a), we notice that the effect of the magnetic parameter is insignificant on the temperature distribution. It is apparent from Fig. 5(b) that an increase in the heat generation parameter α leads to an increase of temperature. This result qualitatively agrees with expectation, since the effect of internal heat generation is to increase the rate of energy transport to the fluid, thereby increasing the temperature of fluid.

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