DEVLOPMENT AND STUDY OF A MATHEMATICAL MODEL USED IN THE AUTOMATIC CONTROL SYSTEMS OF SMALL-SIZE UAVS

M.Sc. Kambushev M. PhD.[1], M.Sc. Biliderov S. PhD.[1]
Faculty of Aviation, Dolna Mitropolia – National Military University, Veliko Turnovo, Bulgaria [1]

m_kambushev@yahoo.com, biliderow_ss@yahoo.com

Abstract: *This paper presents the process of developing and studying of a complete mathematical model of the spatial movement of drones. The model synthesized is used in their automatic control systems.*

Keywords: *mathematical model, small-size uavs, automatic control systems*

РАЗРАБОТВАНЕ И ИЗСЛЕДВАНЕ НА МАТЕМАТИЧЕН МОДЕЛ, ИЗПОЛЗВАН В СИСТЕМИТЕ ЗА АВТОМАТИЧНО УПРАВЛЕНИЕ НА МАЛОГАБАРИТНИ БЕЗПИЛОТНИЛЕТАТЕЛНИ АПАРАТИ

маг. инж. Камбушев М., д-р, маг. инж. Билидеров С., д-р [1]
Факултет "Авиационен", Долна Митрополия – Национален Военен Университет, Велико Търново, България [1]

m_kambushev@yahoo.com, biliderow_ss@yahoo.com

Abstract: *В настоящият доклад е разработен и изследван пълен математичен модел на пространственото движение на безпилотни летателни апарати. Синтезираният модел се използва в системата му за автоматично управление.*

Keywords: *математичен модел, малогабаритни безпилотни летателни, системи за автоматично управление*

1. Introduction

Small-sized UAVs are increasingly used in human activity. This application extends from hobby activities to the performance of professional duties. To enable these aircraft to perform successfully the tasks for which they are intended it is necessary to manage their attitude in space and relative to the ground. This management is carried out thanks to information-managing complex placed aboard a the flying machine.

Management effectiveness of this type of drones is directly connected with both mathematical model that relies on algorithms ruling board computers and sensors necessary to provide this mathematical model with information about the flight.

In popular literature, which describes the dynamics of flight, equations of motion are displayed in the global coordinate system without considering the movement of small-sized unmanned aircraft to a local coordinate system related to the earth's surface. On the other hand, in the literature that describes the different navigation algorithms are reported mostly kinematic relationships without knowing the forces and moments that cause changes of speeds and accelerations of the aircraft. In fact these accelerations are measured on board the aircraft.

This leads to the idea of developing a full mathematical model of spatial movement of aircraft in terms of information tools that are placed on board. Hence the task of the study because it is necessary to examine the applicability of this model in the compact drones.

Depending on the design scheme (airplane or helicopter), which will be used to solve a specific task it is necessary for the equations in the model to undergo certain changes, namely to meet the requirements of each particular scheme. All this is directly related to the algorithm based on the laws for managing small-scale aircraft and copters.

These laws of control are implemented in the automatic control systems of small-sized aircraft, which are responsible for the spatial movement and trajectory of uavs.

2. Synthesizing mathematical model of the flight

In order to derive the equations of the mathematical model of spatial movement of small-sized aircraft some simplifications are made. These simplifications are related to the shape of the Earth, obtaining of the vertical over the location of the aircraft, the fuselage structure, etc.

In the first approximation the shape of the Earth is assumed to be a sphere with radius $\vec{r}$ (Fig. 1), to which the geocentric coordinate system and its vertical are connected. This acceptance simplifies many calculations because it renders it unnecessary to calculate both the radiuses of the curvature of the ellipsoid. Moreover, the global satellite navigation systems such as GPS and inertial navigation systems measure travel speeds and coordinates of the location of the aircraft in the global coordinate system.

In terms of construction, small-sized UAVs are considered rigid bodies with six degrees of freedom.

The equations for spatial movement of miniature unmanned aircraft are displayed with the adoption of the above assumptions, the movement itself is split into movement of the center of mass and movement around the center of mass.

The movement of the center of mass of miniature unmanned aircraft in inertial coordinate system is described by the vector equation of Newton's second law:

$$m.\vec{a} = \sum \overrightarrow{F} \qquad (1)$$

As the accelerations $\vec{a}$ act in inertial coordinate system, where the amount forces $\sum \overrightarrow{F}$ are deployed along the axes of the body-fixed , wind and local frames all of them have to be brought into the body-fixed frame.

Information on the accelerations measured on board the small-sized aircraft is taken from the platform $\vec{\xi}\vec{\eta}\vec{\zeta}$ (Fig. 1), which accommodates the accelerometers and gyros of the inertial navigation system. When on board the aircraft is put a strapdown inertial navigation system then that is a computing platform.

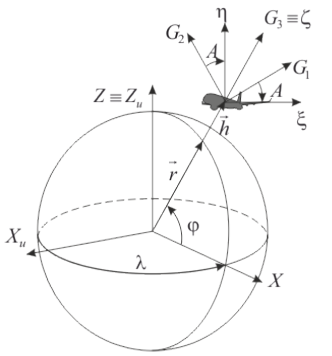

Fig. 1 *Coordinate systems used to navigation purpouse.*

Thus, orienting the axes of the platform in the cardinal directions and the local vertical (Fig.1) on board small-sized drones information is received about the course and the horizon. Accelerations measured by the accelerometers designed are projected onto the axes of the platform.

The accelerometers placed on the platform measure acceleration equal to the difference between the absolute and the relative ground projected onto the axes of the platform $\vec{g}_{\xi\eta\zeta}$:

$$\vec{a}_{\xi\eta\zeta} = \vec{a}^{\,a}_{X_uY_uZ_u} - \vec{g}_{\xi\eta\zeta} \tag{2}$$

Bearing on board the readings of the accelerometers located along the axes of the platform of the inertial system and adding these indications to the projections of the relative acceleration of gravity on the axes of the platform to give we obtain the value of the absolute acceleration of the center of the platform. For obtaining a solution of an equation (1) this acceleration (2) must be designed along the axes of an UAV's body-fixed frame.

Vector equation of the absolute acceleration of the center of mass of small-sized aircraft designed on the axles of the body-fixed frame has the form:

$$\frac{d\vec{V}}{dt} = \frac{\tilde{d}\vec{V}}{dt} + \vec{\omega} \times \vec{V} \tag{3}$$

where with $\frac{\tilde{d}\vec{V}}{dt}$ is marked the acceleration measured in the body-fixed frame and the member $\vec{\omega} \times \vec{V}$ describes the angular velocity of rotation of the body-fixed frame related to inertial frame.

The absolute acceleration (3) is proportional to the sum of the external forces acting along the axes of the body-fixed frame. For different types of small-size aircraft that amount has a different kind.

In the event that a small-sized aircraft is performed constructed in a plane scheme, then the resultant of external forces acting on the axes of the body-fixed frame on the aircraft is described by the expression:

$$\sum \vec{F} = \vec{R} + \vec{P} + \vec{G} \tag{4}$$

where the sum forces $\vec{F}$ is:

- $\vec{P}$ - thrust engine.
- $\vec{G}$ - force on the weight.
- $\vec{R}(\vec{X}, \vec{Y}, \vec{Z})$ - aerodynamic forces.

It is assumed that the motor axis coincides with the axis (*Mx1*) of the body-fixed frame. In this case, this force will be a projection only along the longitudinal axis of the airplane.

Gravity is always directed along local vertical in the case of geocentric coordinate system to the center of the Earth. And projections on the axes of the aircraft are set by the following vector:

$$\begin{pmatrix} G_x \\ G_y \\ G_z \end{pmatrix} = \mathbf{C}^T \begin{pmatrix} 0 \\ 0 \\ -G \end{pmatrix} = \begin{pmatrix} -G\sin\vartheta \\ -G\cos\gamma\cos\vartheta \\ G\sin\gamma\cos\vartheta \end{pmatrix} \tag{5}$$

The aerodynamic forces acting on the aircraft developed on airplane scheme in the most general case are: $\vec{X}_a$ - drag force; $\vec{Y}_a$ - lift force and $\vec{Z}_a$ - side force.

They are created by the air flow and are located in the wind frame. Projections in the body-fixed frame are obtained by means of the rotation matrix $\mathbf{Q}_{(\alpha,\beta)}$:

$$\begin{pmatrix} R_x \\ R_y \\ R_z \end{pmatrix} = \mathbf{Q}_{(\alpha,\beta)} \begin{pmatrix} -X_a \\ Y_a \\ Z_a \end{pmatrix} \tag{6}$$

The case for airplane scheme has been developed in great detail in most of the textbooks on dynamics of flight [1,7,11].

For copter schemes and in particular for tricopter, external forces are projected along axes *Mx1y1z1*- of the body-fixed frame and obtain the equations that define the forward movement of the machine:

$$\sum \vec{F} = \begin{bmatrix} \sum \vec{F}_x \\ \sum \vec{F}_y \\ \sum \vec{F}_z \end{bmatrix} = \begin{bmatrix} -G.\sin(\vartheta) \\ F_A + F_B + F_C.\cos(\alpha) - G.\cos(\gamma).\cos(\vartheta) \\ F_C.\sin(\alpha) + G.\sin(\gamma).\cos(\vartheta) \end{bmatrix} \tag{7}$$

where: F_A, F_B, F_C is the thrust of the engine-propeller system for each of the tricopter's arms; ϑ, γ, ψ are the angles of roll, pitch and yaw; α is the angle of deflection of the tail rotor.

The spatial movement of a tricopter is described in [9] [10].

When considering the movement of quadrocopter (Fig. 2), for the forces acting in flight in its forward movement is obtained the expression:

$$\sum \vec{F} = \begin{bmatrix} \sum \vec{F}_x \\ \sum \vec{F}_y \\ \sum \vec{F}_z \end{bmatrix} = \begin{bmatrix} -G.\sin(\vartheta) \\ F_A + F_B + F_C + F_D - G.\cos(\gamma).\cos(\vartheta) \\ G.\sin(\gamma).\cos(\vartheta) \end{bmatrix} \tag{8}$$

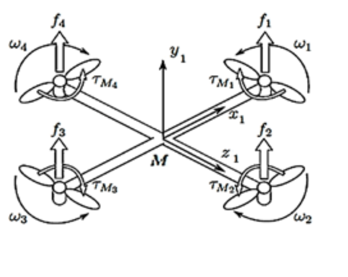

Fig.2 *The forces and the moments in quadrocopter flight.*

So with the help of equation (4) is described the movement of the center of mass of small-sized unmanned aircraft.

The description of the movement around the center of mass of this type of aircraft is carried out by integrating the derivative of the kinetic moment:

$$\frac{d\vec{K}}{dt} = \sum \vec{M} \qquad (9)$$

where $\vec{K} = \sum\left(\vec{r}_k \times m_k \vec{V}_k\right)$ is the kinetic moment of the aircraft.

When switching to the axes of the body-fixed frame with the solution of equation (9) are obtained the projections of the absolute angular velocities of a small-sized unmanned aircraft from (3) on these axes.

Using the so obtained angular velocities is calculated the spatial angular position of the aircraft relative to the natural horizon and the vertical [6]. Actually, the measure angular speeds of the aircraft along the axes of the body-fixed frame on the platform are measured $\vec{\xi}\vec{\eta}\vec{\zeta}$ with the help of , angular speed sensors (gyroscopes).

If the platform is deflected at an angle of azimuth A of the coordinate system $OG_1G_2G_3$ looming cardinal directions (Fig. 3), then the relative angular speeds of the platform is available through the angle of azimuth and angular velocities of the basis relative to the spherical surface of the Earth:

$$\begin{aligned} \omega_\xi &= \omega_{G_1}.\cos(A) - \omega_{G_2}.\sin(A) \\ \omega_\eta &= \omega_{G_1}.\sin(A) + \omega_{G_2}.\cos(A) \\ \omega_\zeta &= \omega_{G_3} \end{aligned} \qquad (10)$$

Unlike mounted platform accelerometers, which measure the absolute acceleration, gyroscops measure the sum of the angular velocity of the platform and the small-size unmanned aircraft.

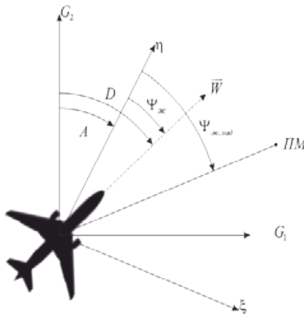

Fig. 3 *Determination of the angular position of the platform* $O\xi\eta\zeta$.

For an airplane scheme, the amount of moments (9) is :

$$\begin{aligned} \dot{\omega}_x &= \frac{1}{I_x}\left[M_x - \left(I_z - I_y\right)\omega_y\omega_z\right], \\ \dot{\omega}_y &= \frac{1}{I_y}\left[M_y - \left(I_x - I_z\right)\omega_x\omega_z\right], \\ \dot{\omega}_z &= \frac{1}{I_z}\left[M_z - \left(I_y - I_x\right)\omega_x\omega_y\right]. \end{aligned} \qquad (11)$$

In case of tricopters for to the equations of the moments along the axes *Mx1y1z1-* of the trikopter's body-fixed frame are obtained ratios ratios:

$$\begin{aligned} &\sum M_x = \left(F_A - F_B\right).\sin\left(\beta/2\right).d; \\ &\sum M_y = F_C.\sin(\alpha).d - \left(M_{A_{yp}} f(F_A) + M_{B_{yp}} f(F_B) + M_{C_{yp}} f(F_C, \cos(\alpha))\right); \\ &\sum M_z = \left(F_A + F_B\right).d.\cos\left(\beta/2\right) - F_C.\cos(\alpha).d + M_{C_{zp}} f(F_C.\sin(\alpha)). \end{aligned} \qquad (12)$$

where: $M_{A_{yp}}, M_{B_{yp}}, M_{C_{yp}}, M_{C_{zp}}$ are reactive moments created by the propellers and projected along the arms of the body-fixed frame; β is the angle between the arms of tricopter; *d* is the length of the arm of tricopter.

In the case of quadrocopter projections of the moments on the body-fixed frame are:

$$\tau_B = \begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix} = \begin{bmatrix} lk(-\omega_2^2 + \omega_4^2) \\ \sum_{i=1}^{4} \tau_{M_i} \\ lk(-\omega_1^2 + \omega_3^2) \end{bmatrix} \qquad (13)$$

where: k is the proportionality coefficient between engine speed and the thrust created by the motor, *l* is the distance between the rotor and the center of mass of quadcopter, *ωi* is the angular speed of the rotor *i*, *τMi* is reactive moment around the axis of the rotor.

Finding the coordinates of the center of mass of the aircraft is done with inertial navigation system or with GPS. The route points of unmanned aircraft are set on the map with geographic coordinates. When measuring coordinates to GPS, are obtained the coordinates in global geocentric coordinate system. To transfer to a local Cartesian system which is suitable for measuring the location of small-size UAVs is necessary: to know the starting point of the flight (B_0, L_0, h_0), and then to perform the transition (B_0, L_0, h_0)→(x_0, y_0, z_0)→(φ_0, λ_0, R).

Via a matrix of directing cosines transfers the fixed-body frame to the basic $(x_1, y_1, z_1) \xrightarrow{C(\psi_{sc}, \gamma, \vartheta)} (G_1, G_2, G_3)$, and then trought (14) are calculated the coordinates in the local system.

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -\sin(\varphi).\cos(\lambda) & \sin(\lambda) & \cos(\varphi).\cos(\lambda) \\ -\sin(\varphi).\sin(\lambda) & \cos(\lambda) & \cos(\varphi).\sin(\lambda) \\ \cos(\varphi) & 0 & \sin(\varphi) \end{bmatrix} . \begin{bmatrix} G_1 \\ G_2 \\ G_3 \end{bmatrix} \qquad (14)$$

Returning in the geographic system is carried out with the transition described in [2]. When the coordinates are measured by the inertial navigation system it has to perform a double integration of the absolute acceleration, and the transitions between the coordinate systems are the same as those described above.

3. Simulation of synthesising model

After finding the mathematical expression for the spatial movement of small-sized unmanned aircraft it is solved inverse problem – What will the instrumental systems installed on board show?

In simulation are used equations for quadrocopter because other are addressed to a airplane diagram [7] and [11], and for tricopter in [9] and [10].

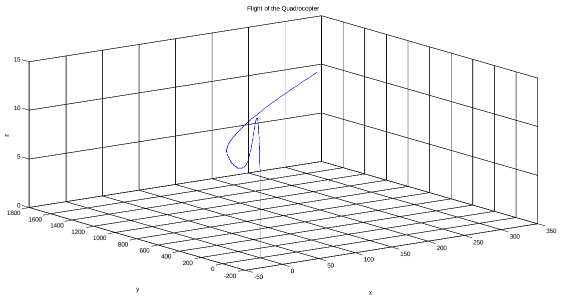

Fig.3 *The flight of the quadrocopter.*

Quadcopter climbs to 10 meters and carry out a straight flight for up to 55 sec., when it receives a signal for change of the heading (Fig. 3).

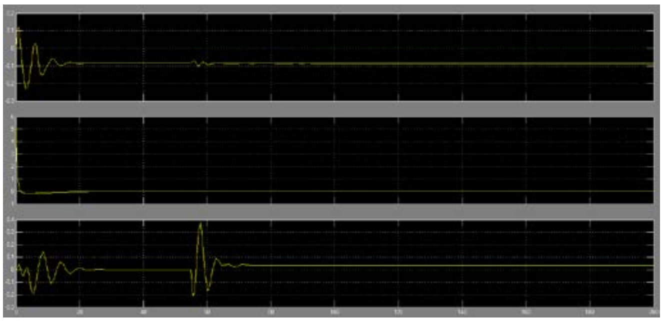

Fig.4 *The measurements of the accelerometers .*
Fig.4 *The measurements of the accelerometers .*

The readings of accelerometers on board without noise in the measurement are shown in Fig.4.

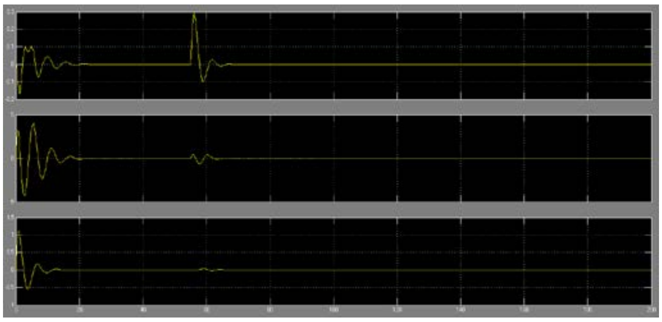

Fig.5 *The measurements of the gyros.*

On Fig.5 and Fig.6 are shown gyroscopic readings and calculated angles of yaw, roll and pitch.

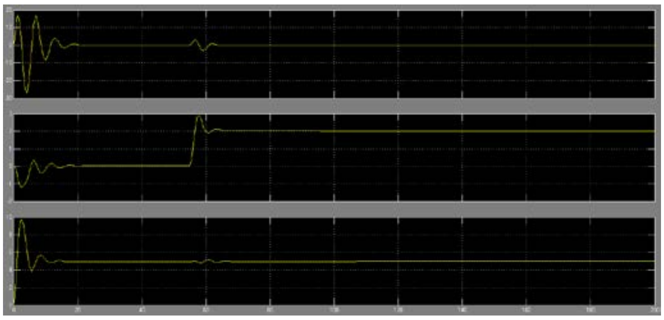

Fig.6 *The Euler's angles.*

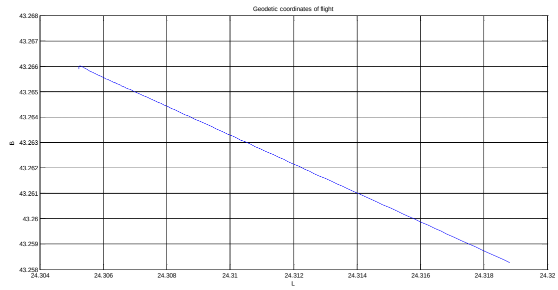

Fig.7 *The geographical coordinates of flight.*

4. Conclusions and results

1. Different models of small-size UAVs are examined in terms of the measurement information;
2. A fix of information from the global navigation systems to local coordinate system is made;
3. The proposed algorithms are used for navigation and management of small-size UAVs;

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