STABILITY OF NONLINEAR AUTONOMOUS SYSTEMS WITH TWO DEGREES OF FREEDOM. AN ANALYTICAL STUDY

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Abstract: *The Thomson-Tait-Chetayev theorem states that "if a system with an unstable potential energy is stabilized with gyroscopic forces, then this stability is lost after the addition of an arbitrarily small dissipation". The importance of this property in growing number of physical examples and engineering applications in the practice is not well good unified and understood, i.e. the destabilizing effect of dissipation needs to be compensated in various gyroscopic devices by applying accelerating forces.*

In the present paper an analytical study of the stability behaviour of a specific class of nonlinear autonomous dynamic systems (i.e. RHS of the equations is a square polynomial) with two degrees of freedom is developed. Considering the general case, we find that the system is multi-stationary and has several possible equilibria. The system is investigated with analytical tools coming from Lyapunov-Andronov theory, and our analytical calculations predict that soft (reversible) loss of stability takes place.

Keywords: STABILITY ANALYSIS, GYROSCOPIC SYSTEMS, ANALYTICAL STUDY

1. Introduction

A system is called autonomous if it explicitly not depends on time. The division of the dynamical systems into autonomous and non-autonomous is in a certain sense conventional. If we consider the values $x_1, x_2, ..., x_n$ as coordinates of point x in the *n*-dimensional space, then we can represent geometrically the state of the dynamical system by means of this point x. Then, x is called a phase point, and the space – *phase space* of the dynamical system.

All dynamical systems can be separated into two main groups: conservative and dissipative. In case that the system is described by autonomous ordinary differential equations then it can be shown (according to the divergence theorem) that the variation of its phase volume dV during a time dt is

$$dV = dt\int\left(\frac{d\dot{x}_1}{dx_1}+\frac{d\dot{x}_2}{dx_2}+...+\frac{d\dot{x}_n}{dx_n}\right)dx_1dx_2...dx_n = dt\int div\dot{x}dx, \quad (1)$$

where $\dot{x}$ is a vector with components $\dot{x}_1, \dot{x}_2, ..., \dot{x}_n$. Hence, the sufficient condition for the conservation of the phase volume has the form

$$div\,\dot{x} = 0. \quad (2)$$

Similarly, the sufficient condition for the decrease of the phase volume is

$$div\,\dot{x} < 0. \quad (3)$$

In nature all systems are dissipative [1], yet if dissipation is very small (for a limited time) then such systems behave as conservative. Dissipative systems can also be sub-classified into passive and active ones.

The study of the stability of the movement of mechanical systems under the action of various forces has a long history. Until now, significant results were obtained for autonomous systems; in [2, 3] the theorems of Thomson (Kelvin) -Tait-Chetayev are used. Practically, the stability of non-autonomous systems influenced by gyroscopic and dissipative forces is not studied well [4].

Linear conservative gyroscopic systems have the form:

$$M\ddot{x} + G\dot{x} + Kx = 0, \quad \text{or} \quad \left(\ddot{x} + \frac{G}{M}\dot{x} + \frac{K}{M}x = 0\right), \quad (4)$$

where the vector x represents the generalized coordinates, M is the mass matrix, G describes the gyroscopic forces and K potential forces. Also, M, G and K are real $n \times n$ matrices with $M^T = M > 0$ (positive definite), $G^T = -G$ (a skew-symmetric matrix) and $K^T = K$ (symmetric matrix). The linear equation (4) is typical for small oscillations of a dynamical system in the region of an equilibrium point ($x = 0$). The results on the problem of the stability of equilibrium can be found in [5].

It is well-known that gyroscopic forces can stabilize the unstable conservative systems, while they cannot destabilize a stable conservative system. In [2, 6], is shown that an unstable conservative system

$$M\ddot{x} + Kx = 0, \quad K \geq 0, \quad (5)$$

can be stabilized by gyroscopic forces if and only if the number of unstable degrees of freedom is even. Hence, when $K < 0$, then the dimension n must be even. Later, for this case in [7], Lakhadanov obtained that suitable stabilizing matrices are $G = g_0G_0$, where $\det G_0 \neq 0$ and g_0 is a sufficiently large number.

There has been a large amount of recent interest in the investigation of gyroscope dynamics. The gyroscope has attributes of great utility to navigational and aeronautical engineering, biology, optics, etc. [8-10, 12]. Different types of gyroscopes (with linear or nonlinear damping, fluid, etc.) are investigated for predicting dynamic responses such as regular and chaotic motions [10, 11].

In mechanics and control theory the problem of force influences over the dynamics of stationary (autonomous) and unstationary systems is important. We will employ potential forces F_p, dissipative forces F_d and gyroscopic forces F_g. The first two forces take care of convergence to the target point and the gyroscopic force handles the obstacle avoidance [13]. Mathematically, the three forces F_p, F_d and F_g can be written in the following form:

$$F_p = -\nabla U(q), \quad F_d = -D(q, \dot{q})\dot{q}, \quad F_g = S(q, \dot{q})\dot{q}, \quad (6)$$

where U is a (potential) function, the matrix D is symmetric and positive-definite, the matrix S is skew-symmetric, q is the position and $\dot{q}$ is a velocity vector.

Gyroscopic forces have two useful perspectives in the dynamics of mechanical systems: (i) they create coupling between different degrees of freedom, just like mechanical couplings; (ii) they rotate the velocity vector just like magnetic field acting on a charged particle. The first interpretation regards the matrix S in (6) as an interconnection matrix and the second interpretation considers S as an infinitesimal rotation. Note that gyroscopic forces are very useful in the stabilization of dynamical systems, because they are unpotential forces with zero power.

According to the Thomson (Lord Kelvin)-Tait-Chetayev and Merkin theories in many physical and engineering applications the destabilizing effect of dissipation needs to be compensated in various gyroscopic devices by applying accelerating forces. In this connection, it is well-known that the equations of perturbed motion of a system have the form

(7) $$\frac{d}{dt}\frac{\partial T}{\partial \dot{q}_l}-\frac{\partial T}{\partial q_l}=-\frac{\partial \Pi}{\partial q_l}+D_l+\Gamma_l+R_l \qquad (l=1,2,...,n),$$

where T is the kinetic energy of the system; Π is the potential energy; q_l and $\dot{q}_l$ are the generalized coordinates and velocities; D_l, Γ_l and R_l are the dissipative, gyroscopic and nonconservative positional forces.

In this paper, the problems associated with analytical investigating stability of equilibriums in a general nonlinear system with two degrees of freedom (i.e. $l=2$) are considered. It is assumed that the forces in the right-hand side of (7) are nonlinear (their expansion in powers of q and $\dot{q}$ is from second order), i.e.

(8) $$J^{(2)}=a_1q_1q_2+a_2q_1\dot{q}_1+a_{23}q_1\dot{q}_2+a_4q_1^2+a_5q_2\dot{q}_1+a_6q_2\dot{q}_2+ \\ +a_7q_2^2+a_8\dot{q}_1\dot{q}_2+a_9\dot{q}_1^2+a_{10}\dot{q}_2^2,$$

where a_1 to a_{10} are positive or negative parameters.

Hence, for $l=2$, the system (7) has the form

(9) $$\ddot{q}_1-d_1\dot{q}_2+(\pm\rho_1^2)q_1=c_1^{(1)}q_1q_2+c_2^{(1)}q_1\dot{q}_1+c_3^{(1)}q_1\dot{q}_2+c_4^{(1)}q_1^2+ \\ +c_5^{(1)}q_2\dot{q}_1+c_6^{(1)}q_2\dot{q}_2+c_7^{(1)}q_2^2+c_8^{(1)}\dot{q}_1\dot{q}_2+c_9^{(1)}\dot{q}_1^2+c_{10}^{(1)}\dot{q}_2^2, \\ \ddot{q}_2+d_2\dot{q}_1+(\pm\rho_2^2)q_2=c_1^{(2)}q_1q_2+c_2^{(2)}q_1\dot{q}_1+c_3^{(2)}q_1\dot{q}_2+c_4^{(2)}q_1^2+ \\ +c_5^{(2)}q_2\dot{q}_1+c_6^{(2)}q_2\dot{q}_2+c_7^{(2)}q_2^2+c_9^{(2)}\dot{q}_1^2+c_{10}^{(2)}\dot{q}_2^2,$$

where $c_i^{(1)}$, $c_i^{(2)}$ $(i=1,...,10)$ and ρ_1, ρ_2 are constants, and q_1, q_2 are normal coordinates.

Let us denote

(10) $$y_1=q_1,\ y_2=\dot{q}_1,\ y_3=q_2,\ y_4=\dot{q}_2.$$

After substitution of (10) into (9) the two second-order ordinary differential equations (9) are reduced to four first-order differential equations in the form

(11) $$\dot{y}_1=y_2, \\ \dot{y}_2=-(\pm\rho_1^2)y_1+d_1y_4+c_1^{(1)}y_1y_3+c_2^{(1)}y_1y_2+c_3^{(1)}y_1y_4+ \\ +c_4^{(1)}y_1^2+c_5^{(1)}y_2y_3+c_6^{(1)}y_3y_4+c_7^{(1)}y_3^2+c_8^{(1)}y_2y_4+ \\ +c_9^{(1)}y_2^2+c_{10}^{(1)}y_4^2, \\ \dot{y}_3=y_4, \\ \dot{y}_4=-d_2y_2-(\pm\rho_2^2)y_3+c_1^{(2)}y_1y_3+c_2^{(2)}y_1y_2+c_3^{(2)}y_1y_4+ \\ +c_4^{(2)}y_1^2+c_5^{(2)}y_2y_3+c_6^{(2)}y_3y_4+c_7^{(2)}y_3^2+c_8^{(2)}y_2y_4+ \\ +c_9^{(2)}y_2^2+c_{10}^{(2)}y_4^2.$$

The aim of the work is to outline the main stability aspects of the system (11). Thus, a qualitative analysis of the system equations is performed in Section 2.

2. Qualitative analysis

In this section, we consider the system (11), which presents an autonomous nonlinear dynamical model. All constants of this model are real and can be negative or positive.

It is easy to see that the system (11) is multi-stationary and has several possible equilibria. The fixed points of the system, $\bar{E}=(\bar{y}_1, \bar{y}_2, \bar{y}_3, \bar{y}_4)$, represented by Eq. (11) can be analytically estimated and are defined by the following set of algebraic equations, including the constants of the model

(12) $$\bar{y}_1=\bar{y}_2=\bar{y}_3=\bar{y}_4=0, \quad \text{if } \bar{y}_1=0,$$

(13) $$\bar{y}_2=\bar{y}_4=0,\ \bar{y}_1\neq 0,\ \bar{y}_3\neq 0, \quad \text{if } \bar{y}_1\neq 0.$$

Investigation of the first fixed point (12)

In order to investigate the character of the first fixed point (Eq. (12)) we make the following substitutions into (11)

(14) $$y_1=\bar{y}_1+x_1=x_1,\ y_2=\bar{y}_2+x_2=x_2, \\ y_3=\bar{y}_3+x_3=x_3,\ y_4=\bar{y}_4+x_4=x_4.$$

Then, after accomplishing some transformations the system (11) has the same form but with variables x_1, x_2, x_3 and x_4. It is seen that the system (11) (for x_1, x_2, x_3 and x_4) enjoys the symmetry, i.e.

(15) $$(x_1, x_2, x_3, x_4)\to(x_1, -x_2, x_3, -x_4).$$

The x_1 and x_3 axes are invariant. All trajectories, which start on the x_1-axis (respectively x_3-axis) remain on it as $t\to\infty$. In this case, the divergence of the flow (11) is

(16) $$D_4=\frac{\partial x_1}{\partial \dot{x}_1}+\frac{\partial x_2}{\partial \dot{x}_2}+\frac{\partial x_3}{\partial \dot{x}_3}+\frac{\partial x_4}{\partial \dot{x}_4}= \\ =(c_2^{(1)}+c_3^{(2)})x_1+(c_8^{(2)}+2c_9^{(1)})x_2+(c_5^{(1)}+c_6^{(2)})x_3+ \\ +(c_8^{(1)}+2c_{10}^{(2)})x_4.$$

The system (11) is dissipative and has an attractor when $D_4<0$.

Following [14], the Routh-Hurwitz conditions for stability of (12) can be written the form

(17) $$p=r=0,$$

(18) $$q=(\pm\rho_1^2)+(\pm\rho_2^2)+d_1d_2>0,$$

(19) $$s=(\pm\rho_1^2)(\pm\rho_2^2)>0,$$

(20) $$R=pqr-sp^2-r^2=0.$$

Here the notations p,q,r,s and R are taken from [14]. The characteristic equation of the system (11) can be written as

(21) $$\chi^4+q\chi^2+s=0.$$

It is seen that in this case the system is structurally unstable because always $R=0$, i.e. the system is always on the boundary of stability. Note here that when $J^{(2)}=0$, then the system (11) is linear conservative.

The basis of stability theory for systems with structurally unstable equilibrium states was developed by Lyapunov [2, 15]. His studies were devoted to various aspects of stability in critical cases, as well as of bifurcation phenomena accompanying the loss of stability at equilibrium states. Here, we mention only the two most common and simple cases, where the characteristic equation of a four-dimensional system

(i) has one zero root; or

(ii) has a pair of complex-conjugated roots on the imaginary axis.

The first case is determined by the condition

(22) $$s=0,\ R>0.$$

Recall that $s=(-1)^4\det A$, where A is the matrix of the linearized system at the equilibrium state. In view of this condition, the equilibrium states associated with the first critical are also called degenerate [15, 16]. Since the implicit function theorem may no longer be applied here, the persistence of such equilibrium state in a neighbouring system is not necessarily guaranteed. Thus, a transition through the stability boundary in the first critical case may result in the disappearance of the equilibrium state. From (17) to (20) it is seen that this case is valid when ρ_1 and/or ρ_2 are zero. But according to (9), we see that ρ_1 and ρ_2 are always different from zero. Hence, we conclude that this critical case is not valid for equilibrium state (12).

The second critical case correspond to

(23) $$R = 0\,,\; s > 0.$$

Here, in contrast to the first critical case, the equilibrium state is preserved in all nearby systems and can only lose its stability. For fixed point (12) this case is valid when $p_1 = \pm\rho_1^2$ and $p_2 = \pm\rho_2^2$ are positive/negative constants.

Investigation of the second fixed point (13)

In this case we have non-simple fixed points. According to [17], a fixed point of a non-linear system is said to be non-simple if the corresponding linearized system is non-simple. Recall that such linear systems contain a straight line, or possibly a whole plane, of fixed points.

The nature of the local phase portrait is now determined by nonlinear terms. Therefore, in contrast to the simple fixed points, there are infinitely many different types of local phase portraits.

There is no detailed classification of non-simple fixed points. However, the definitions of stability (which apply to both simple and non-simple fixed points) provide a classification of qualitative behavior.

In order to investigate the character of the fixed points from kind (13), we make the following substitutions in the system (11)

(24) $$y_i = x_i + \bar{y}_i \qquad (i = 1,\dots,4).$$

Hence, after some transformations system (11) in local coordinates has the form

(25) $$\begin{aligned}
\dot{x}_1 &= x_2,\\
\dot{x}_2 &= m_1x_1 + m_2x_2 + m_3x_3 + m_4x_4 + c_4^{(1)}x_1^2 +\\
&+ c_9^{(1)}x_2^2 + c_7^{(1)}x_3^2 + c_{10}^{(1)}x_4^2 + c_2^{(1)}x_1x_2 + c_1^{(1)}x_1x_3 +\\
&+ c_3^{(1)}x_1x_4 + c_5^{(1)}x_2x_3 + c_8^{(1)}x_2x_4 + c_6^{(1)}x_3x_4,\\
\dot{x}_3 &= x_4,\\
\dot{x}_4 &= m_5x_1 + m_6x_2 + m_7x_3 + m_8x_4 + c_4^{(2)}x_1^2 + c_9^{(2)}x_2^2 +\\
&+ c_7^{(2)}x_3^2 + c_{10}^{(2)}x_4^2 + c_2^{(2)}x_1x_2 + c_1^{(2)}x_1x_3 + c_3^{(2)}x_1x_4 +\\
&+ c_5^{(2)}x_2x_3 + c_8^{(2)}x_2x_4 + c_6^{(2)}x_3x_4,
\end{aligned}$$

where

(26) $$\begin{aligned}
m_1 &= -\left(\pm\rho_1^2\right) + c_1^{(1)}\bar{y}_3 + 2c_4^{(1)}\bar{y}_1,\; m_2 = c_2^{(1)}\bar{y}_1 + c_5^{(1)}\bar{y}_3,\\
m_3 &= c_1^{(1)}\bar{y}_1 + 2c_7^{(1)}\bar{y}_3,\; m_4 = d_1 + c_3^{(1)}\bar{y}_1 + c_6^{(1)}\bar{y}_3,\\
m_5 &= c_1^{(2)}\bar{y}_3 + 2c_4^{(2)}\bar{y}_1,\; m_6 = -d_2 + c_2^{(2)}\bar{y}_1 + c_5^{(2)}\bar{y}_3,\\
m_7 &= -\left(\pm\rho_2^2\right) + c_1^{(2)}\bar{y}_1 + 2c_7^{(2)}\bar{y}_3,\; m_8 = c_3^{(2)}\bar{y}_1 + c_6^{(2)}\bar{y}_3.
\end{aligned}$$

Now, the divergence of the flow (25) is

(27) $$\begin{aligned}
D_4 &= \sum_{i=1}^{n}\frac{\partial\dot{x}_i}{\partial x_i} = m_2 + m_8 + \left(c_2^{(1)} + c_3^{(2)}\right)x_1 +\\
&+ \left(2c_9^{(1)} + c_8^{(2)}\right)x_2 + \left(c_5^{(1)} + c_6^{(2)}\right)x_3 + \left(c_8^{(1)} + 2c_{10}^{(2)}\right)x_4 < 0,
\end{aligned}$$

and the system (25) has an attractor.

According to [14], the Routh-Hurwitz conditions for stability of (13) can be written in the form

(28) $$\begin{aligned}
p &= -\left(m_2 + m_8\right) > 0,\\
q &= m_2m_8 - m_1 - m_7 - m_4m_6,\\
r &= m_1m_8 + m_2m_7 - m_3m_6 - m_4m_5 > 0,\\
s &= m_1m_7 - m_3m_5 > 0,\\
R &= pqr - sp^2 - r^2 > 0.
\end{aligned}$$

When the last two conditions in (28) are not valid, the steady state(s) (13) becomes unstable. The characteristic equation of the system (25) can be written as

(29) $$\chi^4 + p\chi^3 + q\chi^2 + r\chi + s = 0.$$

The stability of a steady state (13) depends on the real part of the roots of the characteristic equation (29). If all roots are negative then the equilibrium state is stable. If at least one root is positive, then the steady state is unstable.

According to [14-16], the conditions $R = 0$ and $s = 0$ are the boundaries of stability. In the boundary of stability $s = 0$, i.e. $m_1m_7 = m_3m_5$, the characteristic equation (29) has one root equal to zero, and the type of the other roots is determined by the expression

(30) $$\Omega = 27r^2 - 18pqr + 4q^3 + 4p^3r - p^2q^2.$$

Thus, we have two cases:

1. If $\Omega < 0,\; p > 0,\; q > 0,\; r > 0,\; R > 0$ and $s = 0$, then the equation (29) has one root equal to zero and three negative real roots;

2. If $\Omega > 0,\; p > 0,\; q > 0,\; r > 0,\; R > 0$ and $s = 0$, then the equation (29) besides one zero root also has one negative root and two complex conjugate roots with negative real parts.

As we mentioned earlier, a transition through the stability boundary $s = 0$ may result in disappearance of the equilibrium state. The system is structurally unstable (un-robust) and through bifurcation it will lose its stability non-reversely. Generally, the stability of mechanical systems, from physical point of view, be connected to gyroscopic forces. However, in this case the gyroscopic devices do not compensate the destabilizing effect of dissipation.

Further, we focus our considerations on the problem of the transition over the stability boundary $s = 0$, i.e. $m_1m_7 = m_3m_5$. Here, we note that for different combinations with values of $\rho_1, \rho_2, c_4^{(1)}$, $c_4^{(2)}, c_7^{(1)}$ and $c_7^{(2)}$, the fourth Routh-Hurwitz condition for stability in (28) can be negative. This question has an immediate significance for the subject of nonlinear dynamics. For stationary regimes, the corresponding problem was solved in [14]. There the boundaries of stability are classified as safe or dangerous – safe boundaries (soft loss of stability) are such that crossing leads to only small quantitative changes of the system's state; dangerous boundaries (hard loss of stability) are such that arbitrarily small perturbations of system beyond them cause significant and irreversible changes in the system's behavior. Generally in accordance with Lyapunov-Andronov theory, the so-called first Lyapunov value $l_1(\lambda_0)$ determines the character (safe or dangerous) of the boundary of stability $s = 0$, when bifurcation parameter λ_0 is slowly changed. Thus, in order to define the type of stability loss of steady state (13) it is necessary to calculate $l_1(\lambda_0)$ on the boundary of stability $s = 0$. In the case of fourth first order differential equations, this value can be determined analytically by the formula in [14]:

(31)

$$l_1(\lambda_0)=\alpha\Big\{a_{11}^{(1)}\sigma_1^2+a_{22}^{(1)}\sigma_2^2+a_{33}^{(1)}\sigma_3^2+\frac{1}{\delta^2}a_{44}^{(1)}(1-\alpha\sigma_1-\beta\sigma_2-\gamma\sigma_3)^2+$$
$$+\frac{2}{\delta}\left(a_{14}^{(1)}\sigma_1+a_{24}^{(1)}\sigma_2+a_{34}^{(1)}\sigma_3\right)(1-\alpha\sigma_1-\beta\sigma_2-\gamma\sigma_3)+$$
$$+2\left(a_{12}^{(1)}\sigma_1\sigma_2+a_{13}^{(1)}\sigma_1\sigma_3+a_{23}^{(1)}\sigma_2\sigma_3\right)\Big\}_1+\beta\{\ldots\}_2+\gamma\{\ldots\}_3+\delta\{\ldots\}_4,$$

where λ_0 is defined as a value of $\rho_1,\rho_2,c_4^{(1)}$, $c_4^{(2)},c_7^{(1)}$ and $c_7^{(2)}$. Here $a_{11}^{(1)}=c_4^{(1)}$, $a_{22}^{(1)}=c_9^{(1)}$, $a_{33}^{(1)}=c_7^{(1)}$, $a_{44}^{(1)}=c_{10}^{(1)}$, $a_{12}^{(1)}=c_2^{(1)},\ldots$ The coefficients $\alpha,\beta,\gamma,\delta,\sigma_1,\sigma_2$ and σ_3 are defined by corresponding formulas presented in [14].

After accomplishing some transformations and algebraic operations for the first Lyapunov value $l_1(\lambda_0)$ (for system (25)), we obtain the main result in this article, i.e. $l_1(\lambda_0)=0$. Note that in (31) now $\alpha=\beta=\gamma=\delta=0$. In other words, the boundary of stability $s=0$ is safe and soft loss of stability take place.

3. Conclusion

In this paper we present an analytical study of the dynamical features of a 4D model describing the gyroscopic dynamics of a general nonlinear system using the Lyapunov-Andronov bifurcation theory. The approach proposed here has a basic advantage, which consists of the following: we can answer the question why the addition (or removal) of some members of the polynomial function $J^{(2)}$ modifies (or does not modify) a system behavior, which would pass (or would not pass) from stable to unstable or from regular to a chaotic one. During the last decade, the robustness (structural stability) has been considered as a key property of mechanical systems [4, 10, 12]. Under this assumption, small changes in the internal or external conditions of a mechanical system are neutralized by the gyroscopic forces which are able to return to the vicinity of the preliminary attractor, while intense changes on the values of parameters characterizing the dissipation can provoke the transition to a new state, potentially a different attractor with its own interval of robustness.

The analytical results lead to the following comments: 1) the general nonlinear system with two degrees of freedom (11) is multi-stationary and has several possible equilibrias; 2) for first fixed point (12) (with coordinates $(0,0,0,0)$), the system is structurally unstable because $R=0$, i.e. the system is always on the boundary of stability; 3) for second fixed point(s) (13), the boundary of stability $s=0$ is safe and soft loss of stability take place. Note that this boundary of stability not depends from gyroscopic forces, as this requirement follows directly from Routh-Hurwitz conditions (28).

To conclude, mathematical modeling and analysis can enable to understand the stabilization mechanism underlying an observed mechanical process, and at the same time, provide a testable hypothesis for future studies. Here, we qualitatively investigate the main properties of the stabilization of a general nonlinear system with two degrees of freedom. Thus, our dynamical predictions can be tested in future numerical simulations.

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