

AN ANALYTICAL MODEL OF THE MOTOR-GRADER MOVEMENT AT PERFORMING WORKING OPERATIONS

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Abstract: A major part of working operations performed by a motor-grader is accompanied by asymmetric action of the resultant vector of external resistance forces applied to the main blade. The eccentrically applied horizontal loads together with additional lateral forces affect the parameters of the machine road-holding ability resulting in deviation of its actual trajectory from the planned one. In the presented work on the basis of the developed model there have been analyzed the impact of operational characteristics of the working process on the indicators of the machine road-holding ability.

Keywords: ROAD-HOLDING ABILITY, PROCESS OF MOVEMENT, MOTOR-GRADER, MATHEMATICAL MODEL.

1. Introduction

A considerable part of research conducted by specialists working in the field of transport, agricultural and earthmoving machinery is devoted to issues of ensuring the road-holding ability of a machine during its movement. Specificity of performing technological operations by different types of machines results in the need for development and analysis of non-standard mathematical models of the machines movement.

2. Analysis of publications

A typical design situation of losing the road-holding ability by transport machines is their movement at cornering. In this case the cause of the machine's deviation from the set movement trajectory is lateral inertia forces, which depend on the mass of the machine, its speed and the corner radius.

The physical processes considered by researchers in this respect are slippage of the machine wheels due to the elastic deformation of the tires and skidding of the wheels of the propelling devices.

To analyze the movement parameters, a dynamic model of the machine movement [1] is usually considered. An example of a dynamic model of the machine movement on the curved part of the trajectory is shown in Table 1.1. For agricultural machines the cause of losing the road-holding ability in the process of performing work operations are most often additional lateral components of the gravity loading, which occur at moving on support surfaces with a transverse gradient. Besides, due to a specific design of the implement, an emergence of lateral components of working resistance forces in the performance of technological operations is possible.

For the analysis of the movement trajectory of machines the researchers propose to analyze dynamic models of plane motion. Since agricultural implements are hinged to the base tractor vehicles and tractors themselves can be articulated aggregates, a special attention in such research is paid to multihinged systems [2]. For example, in [3] there considered the movement of an agricultural machine equipped with a trailing implement (Table 1.2). In compiling a dynamic model of the plane motion of an articulated system, the author took into account the action of lateral forces of working resistance on the implement.

For excavating and earth-moving machines, characteristics of the road-holding ability are influenced, apart from the abovementioned parameters, by certain additional destabilizing factors. Firstly, a number of machines is equipped with active implements, which, during the excavation of soil move relative to the base machine leading to occurrence of additional inertia forces in the horizontal plane. Secondly, the main vector of the external resistance forces acting on the blade- and bucket-type implement can be considerably displaced in relation to the longitudinal axis of

the base machine, which has randomly changed its position in the process of soil digging. This causes not only occurrence of additional torques in the horizontal plane but also additional lateral loads on the implement.

For example, in [4] there considered the road-holding ability of a tracked excavating machine equipped with a rotary implement, which makes fan-like movements at digging a trench (Table 1.3). When analyzing the movement of the machine the author proposes to take into account the horizontal and vertical components of the eccentrically applied force generated by the working equipment.

It is suggested evaluating the road-holding ability of such a machine by the value of the ratio of the road-holding ability:

$$k_{kc} = \frac{M_{op}}{M_p} > 1, \quad (1)$$

where M_{op} — the moment preventing the machine from making a U-turn; M_p — the turning moment

Work [5] analyzes the movement of a motor grader in situations of an intense increase in resistance forces acting on the main blade. The author used a system of integro-differential equations as a basic dynamic model (Table. 1.4). It has been theoretically and experimentally proved that skidding of the wheels of propelling devices and U-turning of the machine can occur at the realization of such processes.

Summarizing the performed review it is possible to make the following conclusions:

1) for evaluating the parameters of road-holding ability of machines an analysis of dynamic models of their movement is usually performed. As a rule, the authors consider a plane-parallel movement of the machine in the horizontal plane not taking into account the support reactions and hence the tractive forces between the elements of the undercarriage;

2) destabilizing factors influencing the road-holding ability of earth-moving machines are the machine movement along a curvilinear trajectory, work on surfaces with transverse gradient, dynamic effects from the active implement, coordinates of the application and direction of the resultant vector of resistance forces on the implement;

3) the road-holding ability, as a rule, is evaluated using a road-holding coefficient, defined as the ratio of the holding forces to turning ones. The loading schemes are presented in a static form.

This approach enables performing a primary express evaluation of characteristics of earth-moving machines but it doesn't take into consideration the fact that for machines with blade- and bucket-type implements the value and position of the resultant vector of external resistance forces changes with the

movement of the machine. Analysis of the characteristics of the road-holding ability in this case, in our opinion, should be based on studying the shape of the machine movement trajectory. To perform such actions can only be possible after investigating the dynamic model of movement of earth-moving machines in the course of the work operations of soil digging.

The purpose and objectives

The purpose of the article is the development and analysis of a dynamic model of motor grader movement in the process of soil digging under the most unfavorable scheme of application of external resistance forces from the side of the developed environment.

Table 1. Parameters characterizing the road-holding ability

Item No.	Author	Dynamic model	Mathematical model
1	D. M. Kleis [1]		$2 \left[\frac{m_a}{2} + \frac{l_K}{2r_a^2} + \frac{l_z}{2R^2} \right] \dot{V}_{x1} =$ $= (m_x \cdot \varphi_x \cdot G - 0,5 \cdot C_x \cdot \rho \cdot F \cdot V_{x1}^2) \frac{h-r_a}{L} \varphi_x (1 - \cos \bar{\alpha}) -$ $- \frac{h-r_a}{L} \varphi_y \sin \bar{\alpha} (m_x \cdot \varphi_x \cdot G - 0,5 \cdot C_x \cdot \rho \cdot F \cdot V_{x1}^2) -$ $- P_{Cx} - \left[\varphi_x (a + b \cos \bar{\alpha}) + \varphi_y b \sin \bar{\alpha} \right] \frac{G}{L};$ $I_x \cdot \ddot{\psi}_{\delta 0} - 0,5 \cdot \mu \cdot B^2 \cdot \dot{\psi}_{\delta 0} - (c_1 + c_2 - m_a g \cdot h_{\delta 0}) \cdot \psi_{\delta 0} =$ $= -P_{Cy} \cdot h_{\delta 0};$ $I_y \cdot \ddot{\beta} + 2 \cdot \mu \cdot (a^2 + b^2) \cdot \dot{\beta} + (c_1 + c_2 - m_a g \cdot h_{\delta 0}) \cdot \beta =$ $= -P_{Cx} \cdot h_{\delta 0}.$
2	N. P. Artemov [3]		$\left. \begin{aligned} m_1 \ddot{\xi} + C_1 \dot{\psi}_1 + D_1 \dot{\psi}_2 + E_1 \dot{\psi}_3 + F_1 &= G_1 \\ m_2 \ddot{\eta} + C_2 \dot{\psi}_1 + D_2 \dot{\psi}_2 + E_3 \dot{\psi}_3 + F_2 &= G_2 \\ C_1 \dot{\xi} + C_2 \dot{\eta} + C_3 \dot{\psi}_1 + F_3 &= G_3 \\ D_1 \dot{\xi} + D_2 \dot{\eta} + D_4 \dot{\psi}_2 + E_4 \dot{\psi}_3 + F_4 &= G_4 \\ E_1 \dot{\xi} + E_2 \dot{\eta} + E_4 \dot{\psi}_2 + E_5 \dot{\psi}_3 &= G_5 \end{aligned} \right\}$
3	A. B. Koval' [4]		$F_z = -\frac{V^2}{R} \left(\sum_{i=1}^{n'} m'_i \cdot \cos[\xi \cdot (i-1) - \sigma] + \sum_{i=1}^{n'} m'_i \cdot \cos(\xi \cdot i - \sigma) \right)$ $F_\sigma = -\frac{V^2}{R} \left(\sum_{i=1}^{n'} m'_i \cdot \sin[\xi \cdot (i-1) - \sigma] + \sum_{i=1}^{n'} m'_i \cdot \sin(\xi \cdot i - \sigma) \right)$

<p>4</p>	<p>A. V. Voronovych [5]</p>		$m \ddot{x} + \frac{\partial}{\partial x} \left[\int_0^{x-\beta h-\gamma b} f_x(x_x) dx \right] = T(\dot{x}) - f(N_z + N_p);$ $m \ddot{z} + C_{zz}(z + \beta l_c) + C_{zp}(z - \beta l_1) - C_z(z - \beta l) + \frac{\partial}{\partial z} \left[\int_0^{x-\beta h-\gamma b} f_x(x_x) dx \right] = -\psi_v R_x;$ $m \ddot{y} + C_{yz}(y + \gamma l_c) + C_{yp}(y - \gamma l_1) + \frac{\partial}{\partial y} \left[\int_0^{x-\beta h-\gamma b} f_x(x_x) dx \right] = -\psi_b R_x;$ $J_0 \ddot{\beta} + C_{zz} l_c (z + \beta l_c) - C_{zp} l_1 (z - \beta l_1) - C_z l (z - \beta l) + \frac{\partial}{\partial \beta} \left[\int_0^{x-\beta h-\gamma b} f_x(x_x) dx \right] = -T(\dot{x})h + f(N_z + N_p)h + \psi_v R_x l;$ $J_0 \ddot{\gamma} + C_{yz} l_c (y + \gamma l_c) - C_{yp} l_1 (y - \gamma l_1) + \frac{\partial}{\partial \gamma} \left[\int_0^{x-\beta h-\gamma b} f_x(x_x) dx \right] = -T(\dot{x})b + f(N_z + N_p)b - \psi_b R_x b.$
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Development of a mathematical model of a solid body plane motion

By road-holding ability we mean the capability of an earth-moving machine to move along a predetermined trajectory within deviation tolerances regulated by construction standards and regulations. To evaluate the parameters of the road-holding ability at the stages of design and improvement of the machine, it is necessary to develop and analyze a dynamic model of its movement.

In contrast to the considered above dynamic models of machine movement enabling the estimation of parameters of the road-holding ability, we propose regarding motor graders to consider the following factors, which have a significant influence on the formation of their movement trajectory during soil cutting operations:

- asymmetric application of the resultant vector of the external load on the main blade;
- redistribution of tractive forces between leading sides of the balance truck during the machine's operation on the sites with a transverse gradient;
- variation of tractive forces caused by the change in the adhesion weight of the motor grader as a result of its unsteady movement. Such cases are possible at the stages of intensive penetration of the blade into the soil at the initial stage of digging, changing the depth of cutting as a result of the control action by hydraulic cylinders of lifting/lowering mechanism of the blade while digging the soil, at the collision of the blade with a rigid obstacle.

In the process of justification and development of the dynamic scheme of a motor grader the following objective factors were taken into account:

1. the motor grader movement occurs on a horizontal surface. The changing of the vertical surface is neglected, as in cases of most operations the wheels of the balance truck move along the surface, which has been already leveled;

2. elastic properties of the tires in the design model are taken into account. It is caused by the fact that their deformation in the vertical plane causes the shift of the machine's center of mass and redistribution of the support reactions, which influences the formation of tractive forces. The deformation of tires in the horizontal plane causes vibration in the frame of the machine in the same plane affecting the formation of characteristics of the motor grader road-holding ability.

3. parameters of the road-holding ability describe the movement of the machine over long distances (from several meters to several tens of meters). That is why the elastic deformations of the working equipment and the bearing metal structure are neglected, given their insignificance in comparison with the general movement of the entire machine.

4. modern motor graders are energy-saturated machines since they are equipped with the internal combustion engine with excess power. At performing work operations of soil digging, when the speeds of machine movement are low and the reduced mass of rotating elements of the transmission and engine flywheel are in the tens and hundreds of times higher than the mass of the motor grader itself, with a change in the value of external resistance forces the number of engine shaft revolutions changes insignificantly. During the experiments there were recorded a decrease in the number of revolutions of the crankshaft no more than by 5%, which is not essential. In the calculations we assume that it remains constant [5].

All the *abovementioned* aspects make it possible to pass to developing a dynamic scheme of a motor grader for performing soil digging operations. Since the experiments have shown that the greatest effect on the road-holding ability is made by the absolute values, direction and coordinates of applying the resultant vector of

external resistance forces [6], the dynamic model considers the worst in terms of road-holding ability situation, when the digging is performed by the blade edge. In this case the blade is sloped in the vertical plane, the portion of the cut soil prism is formed and placed in front of the blade and the other portion goes under the blade from the opposite side.

Taking into account the previously considered simplifications the dynamic scheme of a motor grader will have the form shown in Fig. 1. The machine movement occurs relative to the fixed inertial coordinate system Oxyz. The analysis of the machine trajectory in this coordinate system allows defining key characteristics of the motor grader road-holding ability: the lateral displacement and rotation angle relative to the longitudinal axis [7].

In the process of performing the work operation of digging the driving wheels generate tractive forces T_1 and T_2 . The given scheme of the application of forces is true for motor graders with the most common wheel arrangement $1 \times 2 \times 3$. The rolling resistance forces are respectively W_{f1} and W_{f2} — on the sides of the balance truck and W_{fn} — on the front axle. The resultant vector of resistance to soil digging is presented by two components: R_r — longitudinal component and R_δ — lateral component. Both of these forces cause the emergence of loads, cross-resistance and torque, which could destabilize the trajectory of the motor grader movement. In the areas of contact of propelling devices with the support surface there

act forces of lateral resistance to displacement $P_{\delta 1}$, $P_{\delta 2}$ and $P_{\delta n}$, which values are determined by the wheels adherence to the ground.

On the basis of Lagrange equation the motor grader movement can be described by the following system of differential equations:

$$\begin{cases} m\ddot{x}_1 = T_1 + T_2 - m\dot{y}_1\dot{\varphi} - W_{f1} - W_{f2} - \\ - W_{fn} - R_r \\ m\ddot{y}_1 = m\dot{x}_1\dot{\varphi} - R_\delta - P_{\delta 1} - P_{\delta 2} + P_{\delta n} \\ I\ddot{\varphi} = (T_2 - T_1)\frac{l_6}{2} + (W_{f1} - W_{f2})\frac{l_6}{2} - \\ - (P_{\delta 1} + P_{\delta 2})l_1 + R_\delta l_2 + R_r l_3 - P_{\delta n} l_4 \end{cases} \quad (2)$$

In the given mathematical model m is the mass of the motor grader, I is the moment of inertia of the motor grader relative to its center of mass.

To build the motor grader trajectory in the inertial (fixed) coordinate system Oxy, it is necessary to solve the system of differential equations [8].

$$\begin{cases} \dot{x} = \dot{x}_1 \sin \varphi - \dot{y}_1 \cos \varphi \\ \dot{y} = \dot{x}_1 \cos \varphi + \dot{y}_1 \sin \varphi \end{cases} \quad (3)$$

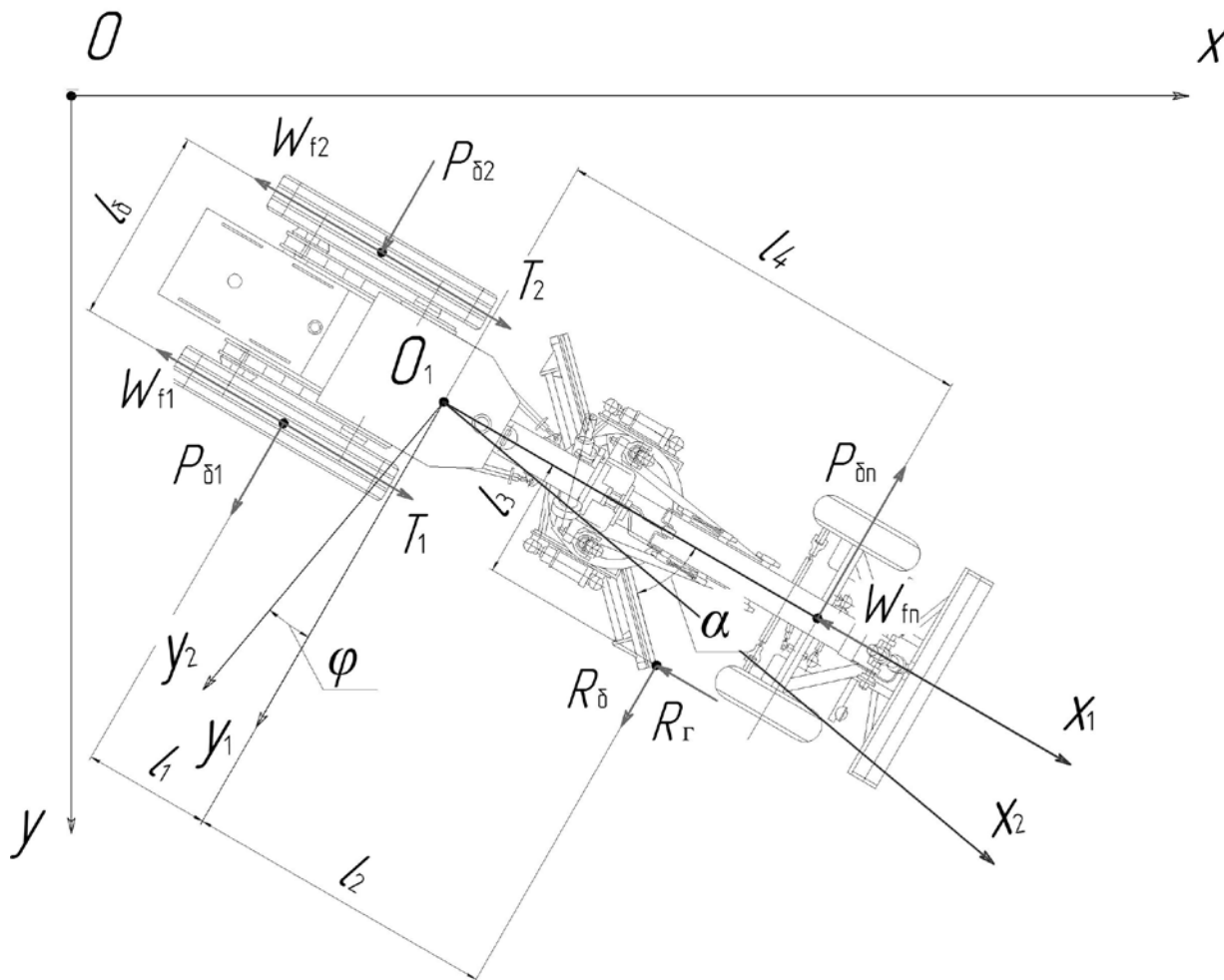


Fig.1 The motor grader dynamic scheme.

The tractive forces on the driving wheels depend on the values of support reactions, theoretical (\dot{x}_{1r}) and actual (\dot{x}) speeds of the machine movement, type of the undercarriage and characteristics of the support surface [9]:

$$\delta = \frac{\dot{x}_{1r} - \dot{x}}{\dot{x}_{1r}} = A \frac{T}{N} + \frac{(1 - A\varphi_{cu})}{\varphi_{cu}^m} \left(\frac{T}{N} \right)^m, \quad (4)$$

where A is an empirically determined coefficient; φ_{cu} is a coefficient of the wheel adherence to the support surface; N is a support reaction on the driving wheel; m is an exponent determined by an experiment.

The value of support reactions on the driving wheels change in unsteady regimes of the machine movement resulting from situations of abrupt change in the area of the cut chips of soil or

collision of the blade with a rigid obstacle. To determine their values, it is necessary to consider the dynamic model of the motor grader in the vertical plane (Fig.2).

The machine movement in this case will be described by the system of differential equations:

$$\begin{cases} m\ddot{x}_1 = T_1 + T_2 - m\dot{y}_1\dot{\phi} - W_{f1} - W_{f2} - W_{fn} - R_r \\ m\ddot{z}_1 + (C_{12} + C_{np})z = 0 \\ I_b\ddot{\beta} + C_{12} \cdot l_1^2\beta + C_{np} \cdot l_2^2\beta + C_n \cdot l_4^2\beta = 0 \end{cases} \quad (5),$$

where C_{12} is the elasticity coefficient of the balance truck tires; C_{np} — reduced coefficient of elastic restraint between the soil and hydraulic drive of the working equipment; C_n — elasticity

coefficient of tires of the front axle wheels; I_b — moment of inertia of the motor grader relative to the center of mass in the vertical plane; z_1, β — additional generalized coefficients

Solution of the system of differential equations (5) allows determining the deformation of the front of the rear tires of the motor grader and calculate the current values of the support reactions:

$$\begin{cases} N_n = \frac{Gl_1}{l_1 + l_4} + (z_1 + l_4\beta)C_n \\ N_1 = \frac{Gl_4}{l_1 + l_4} - (z_1 + l_1\beta)C_{12} + \frac{m\ddot{y}_1 \cdot h_a}{l_6} = 0 \\ N_2 = \frac{Gl_4}{l_1 + l_4} - (z_1 + l_1\beta) - \frac{m\ddot{y}_1 \cdot h_a}{l_6} = 0 \end{cases} \quad (6)$$

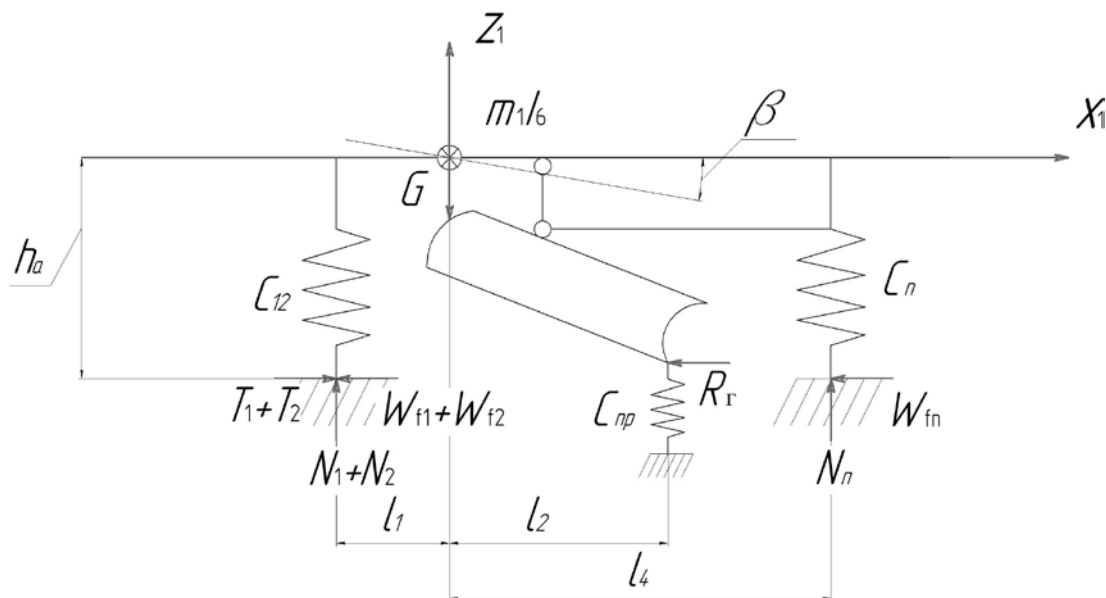


Fig. 2 The calculation model for determining the support reactions on the driving wheels.

This equation can not be solved in its explicit form with respect to the traction force T. The design feature of the vast majority of modern motor graders is the use of torque converter in their transmission. The experiments conducted on the basis of KhNAHU with a motor grader Д3К-251 demonstrated that with this arrangement of the transmission the engine speed decreases by no more than 4.6% even in situations where the blade is locked in the ground. The engine power is sufficient to bring the leading propelling devices to the regime of full slippage. In any case, in order to simplify the calculations when determining the slip coefficient, it can be assumed that the crankshaft speed will be constant at variable external resistance forces. Taking into consideration characteristics of the concurrent working of the engine and hydro-mechanical transmission and the accepted simplifications, the dependence (4) can be approximated and reduced to the form [8].

$$\begin{aligned} T_1(\dot{x}_1) &= N_1\varphi_{cu} [1 - a\dot{x}_1 - b\dot{x}_1^5], \\ T_2(\dot{x}_1) &= N_2\varphi_{cu} [1 - a\dot{x}_1 - b\dot{x}_1^5], \end{aligned} \quad (7),$$

where a and b are approximate coefficients, N_1 and N_2 are support reactions on the respective leading sides.

The forces of rolling resistance depend on the value of support reactions on the grader wheels and type of the support surface. In a general form they can be calculated by the formula:

$$\begin{aligned} W_{f1} &= N_1f, \\ W_{f2} &= N_2f, \\ W_{fn} &= N_n f, \end{aligned} \quad (8)$$

Lateral resistance forces act as retaining restraints. The limit value of these efforts can be determined by the dependence:

$$P_6 = N\varphi_{cu,6}, \quad (9),$$

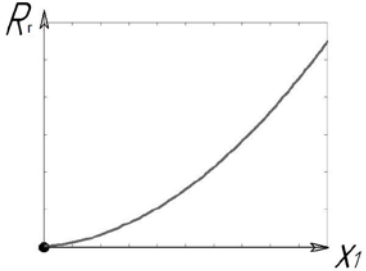
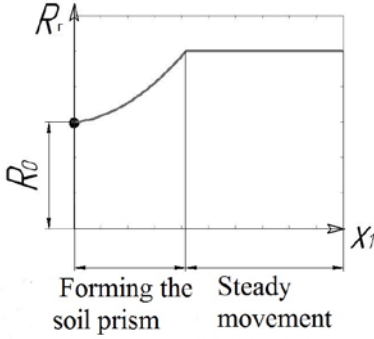
where $\varphi_{cu,6}$ — is a coefficient of adherence of wheels to the support surface in the lateral direction relative to the rolling direction. If we denote the total active lateral forces acting on the support pneumatic tires respectively as $\sum F_1, \sum F_2, \sum F_n$, in analytical form it can be presented as follows:

$$\begin{aligned} P_{61} &= \begin{cases} \sum F_1, \text{ если } \sum F_1 \leq N_1\varphi_{cu,6} \\ N_1\varphi_{cu,6}, \text{ если } \sum F_1 > N_1\varphi_{cu,6} \end{cases}, \\ P_{62} &= \begin{cases} \sum F_2, \text{ если } \sum F_2 \leq N_2\varphi_{cu,6} \\ N_2\varphi_{cu,6}, \text{ если } \sum F_2 > N_2\varphi_{cu,6} \end{cases}, \\ P_{6n} &= \begin{cases} \sum F_n, \text{ если } \sum F_n \leq N_n\varphi_{cu,6} \\ N_n\varphi_{cu,6}, \text{ если } \sum F_n > N_n\varphi_{cu,6} \end{cases}, \end{aligned} \quad (10)$$

The horizontal and lateral components of the digging resistance depend not only on parameters of the working equipment and characteristics of the developed environment but also on the type and method of performing work operations. In a general case the horizontal component is the function of the motor grader movement along the x axis, and for different working situations it can be expressed by the dependences given in Table 2.

In the dependences presented in Table 2 the following letter symbols are used: $R_0 = W_p$ — soil digging resistance; W_o — resistance to the movement of soil up the blade; W_{np} — resistance to the movement of the soil prism in front of the blade; W_B — resistance to the movement of the soil along the blade.

Table 2. The horizontal component

Work operation	Graphic interpretation	Analytical dependence
1	2	3
Intensive deepening of the blade		$R_x = x_1 - b_1x_1 + c_1x_1^2$
Digging from the pit		At the stage of forming the soil prism: $R_x = R_0 - b_1x_1 + c_1x_1^2$ At the state of steady movement (the prism does not increase): $R_x = W_p + W_{np} + W_o + W_B$

Based on the known regularities we can form the following system of dependences

$$\begin{aligned}
 W_p &= Fk \\
 W_{np} &= V_{np} \frac{\delta_{rp}}{k_p} g\mu_1 \\
 W_o &= V_{np} \frac{\delta_{rp}}{k_p} g\mu_1\mu_2 \cos^2\beta \quad (11), \\
 W_B &= V_{np} \frac{\delta_{rp}}{k_p} g\mu_1 \sin\alpha
 \end{aligned}$$

where F is the area of the cut soil chip; k — specific coefficient of soil cutting resistance; δ_{rp} — soil density in natural occurrence; k_p — coefficient of soil softening; $g = 9,8 M/c^2$ — gravitational acceleration; μ_1, μ_2 — respectively coefficients of internal and external soil friction; β — cutting angle; α — attack angle.

The lateral component of the soil digging resistance

$$R_0 = R_r \sin\beta \quad (12)$$

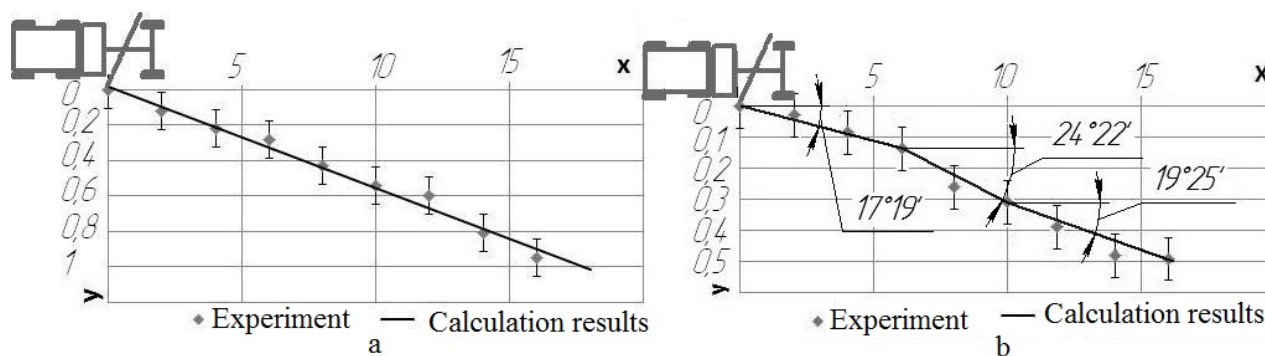


Fig. 3 The movement trajectory of the motor grader in the process of soil digging from the pit

Simultaneous solution of equations (2) and (3) was performed by the numerical Runge-Kutta method with a variable step [7]. The results of the analytical calculations and experimental data obtained during the experiments with the motor grader ДЗК-251 are shown in Fig.3.

The experimental data and calculation results show that as a result of action of destabilizing factors the real motor grader movement trajectory deviates from the planned straight-line one. In the case illustrated by Figure 3 a, the U-turn of the machine occurs at the initial moment of digging. Since the digging is performed from the pit, this moment corresponds to a one-time effect of dynamic loads after which the machine moves steadily along the straight-line trajectory. For the studied machine the similar situations are recorded at the initial speeds of the machine of 0.66 – 1.21 m/s, blade sideshift of 0 – 0.7 m, attack angle of 0 – 6 deg., cutting depth 0.03 – 0.06 m. With the growth of the digging resistance forces, which was determined by increasing the cutting depth to 0.15 m, the trajectory of the machine movements abruptly changes Fig. 3. b. The trajectory consists of straight-line sections, with the joints of the sections being points of the machine U-turning caused by sideways skidding of pneumatics tires. In terms of physics this phenomenon can be explained as follows. In the course of digging operations a prism of soil is formed in front of the blade, which results in the increase in resistance forces. When the forces reach their limit values, the machine stops its linear motion, the driving wheels go into a complete slippage regime and there occurs a lateral displacement of the rare of the motor grader. As the machine rotates, a portion of the soil prism moves under the blade and the total value of resistance forces to straight-line motion reduces enabling the machine to continue its further movement and soil digging. The theoretically calculated trajectories of movement satisfactorily fit the experimental data (Figure 3).

Conclusions

The conducted studies have shown that geometric parameters of the working process determining the implement position in the space at soil digging operations have a decisive effect on the grader road-holding ability. The formation of the machine movement trajectory is determined by the effect of both dynamic loads on the blade and the variable in the direction of the digging resistance force. The proposed motor grader dynamic model enables taking into consideration the abovementioned factors and gives a good fit of the theoretical and experimental data.

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