DISTRIBUTION OF SUBHARMONIC WAVES WITH THE NONLINEAR MECHANISM OF DISSIPATION OF ENERGY

Prof. Dr Tyurekhodjaev A.N.[1], Ass.prof. PhD Sergaziyev M.Zh. [2]
Institute of Industrial Engineering – Kazakh National Technical University after K.Satpaev - Almaty, the Republic of Kazakhstan [1]
Faculty of Information Technologies – International Information Technologies University - Almaty, the Republic of Kazakhstan [2]

Abstract: *The statics and dynamics of mechanical systems with contact dry friction, representing the nonlinear mechanism of the energy dissipation, is a new actual direction in the nonlinear mechanics of deformable solids. In the case of dynamic deformation, the nonlinearity of the dissipation mechanism represents an a priori unknown sign-nonlinearity of the required function speed. Such tasks are considered in hyperbolic type equations for nonlinear system, describing the distribution and attenuation of nonlinear waves. Analytical results for the distribution of nonlinear waves in a system with contact dry friction under the influence of cyclic loads were obtained. The class of loads under which the system shows subharmonic and ultraharmonic oscillations was determined.*

Keywords: NONLINEAR ELASTIC WAVES, DRY CONTACT FRICTION, HYPERBOLIC TYPE EQUATION

1. Introduction

The propagation of nonlinear waves in a mechanical system with dry friction under cyclic loading with a period that is integrally greater or less than the system's natural period of oscillation was investigated analytically.

Problems of this sort reduce to an investigation of nonlinear systems of hyperbolic-type equations and are connected to the investigation of the propagation and attenuation of nonlinear waves [1,2]. Nonlinearity is due to the presence of dry contact friction. In the case of dynamic deformation, the nonlinearity of the dissipation mechanism provides an a priori unknown nonlinear velocity function. In the case of movement, the friction assumes a maximum value with a plus or minus sign; at standstill, it takes any value between its positive or negative maximum.

The main complexity consists of determining the expression of the friction's sign function, which significantly depends on both the boundary and initial conditions, of the law of dry friction. The dependence domain for resolving problems of this sort is determined by the kappa-function method of Nikitin-Turekhodjayev [3]. Applying the kappa-function method in many problems of this sort can determine the nonlinear function of friction and record it as an infinite sum of Heaviside functions with shifted arguments. Then, the nonlinear velocity function becomes a function of independent arguments, and the problem becomes linear. Thus, it can be resolved using one of the standard methods for solving linear equations.

Analytical results were obtained for a class of problems wherein the frequency of the external load is *n* times greater or less than the system's free frequency. The analysis of results obtained for $n = 1; 2; 3; 4$ allowed us to construct solutions on the whole area of the dependence domain of the problem solution ($0 \le t < \infty$, $0 \le x \le \ell$). The general solution of the problem is recorded by progressive waves that covered the travel way. The record of solutions in characteristic regions gives a pictorial view of the functions of displacement, stress and velocity.

2. Nonlinear steady oscillations of a mechanical system with dry friction under cyclic loading

An analytical investigation was performed on the pattern of nonlinear steady wave propagation in mechanical systems caused by the presence of a nonlinear mechanism of energy dissipation. Analytical results were obtained on the longitudinal oscillations of a terminal flexible rod, the surface of which interacted with the environment by Coulomb's dry friction law, during dynamic agitation as a "rectangular harmonic load" with a frequency integrally greater than the system's free frequencies.

The specificity of the investigation consisted of the fact that problems with contact dry friction are incorrect to the extent that nonlinearity does not allow us to generalize the results of one problem into a class of problems. Thus, each type of loading must be investigated separately.

We shall demonstrate the method of solution with an example investigating longitudinal oscillations of a terminal flexible rod under the effect of an oscillatory step load with a frequency three times greater than the system's free frequencies.

Suppose that we have a rod, the end $x = \ell$ of which is plugged, and a periodic stress is applied on the end $x = 0$ (Fig. 1).

(1) $$\sigma(0,t) = \sigma_0 \left\{ H(t) - 2\sum_{k=0}^{\infty} (-1)^k H\left(t - \frac{2}{3} k\ell/a\right) \right\},$$

where $\sigma(x,t)$ is a normal stress on section *x*, $\upsilon(x,t)$ is the velocity and *H(z)* is the Heaviside's unit function.

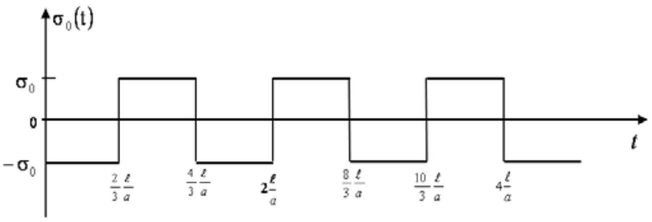

Fig. 1. *Normal stress at the end x=0 of the rod.*

At the initial instant, the rod is assumed to be at rest and unstressed:

(2) $\vartheta = 0$, $\sigma = 0$, $t \le 0$.

The equation of motion and Hooke's law can be generally recorded as follows:

(3) $$\frac{\partial \sigma}{\partial x} = \frac{1}{\rho}\frac{\partial \upsilon}{\partial t} + \aleph\left(\upsilon/|\upsilon| , \partial\upsilon/\partial t \,/|\partial\upsilon/\partial t| \right) \cdot q, \qquad \frac{\partial \sigma}{\partial t} = E\frac{\partial \upsilon}{\partial x},$$

where the sign of the velocity $\aleph = sign(\vartheta)$ if $\vartheta \ne 0$ and $\aleph \in [-1;1]$ if motion occurs with stops; ρ is the density of the material; q is the friction force and E is the elasticity modulus.

The system (3) is highly nonlinear due to presence of the function $\aleph$. The difficulty in solving this nonlinear problem can be overcome because the frictional force, in the investigated problem, being passive, cannot change the sign of the velocity. Counting direct and reflex waves gives the following result for the function $\aleph$:

$$\aleph(x,t)=\sum_{k=0}^{\infty}\left\{H\left(t-x/a-2k\ell/a\right)-H\left(t-x/a-2\left(k+\tfrac{1}{3}\right)\ell/a\right)\right\}$$
$$\left(H(x)-H\left(x-\tfrac{2}{3}\ell\right)\right)+$$
$$+\sum_{k=0}^{\infty}\left\{H\left(t-x/a-2k\ell/a\right)-H\left(t+x/a-2(k+1)\ell/a\right)\right\}\cdot H\left(x-\tfrac{2}{3}\ell\right)-$$
$$-\sum_{k=0}^{\infty}\left\{H\left(t-x/a-2\left(k+\tfrac{1}{3}\right)\ell/a\right)-H\left(t-x/a-2\left(k+\tfrac{2}{3}\right)\ell/a\right)\right\}\cdot$$
$$\cdot\left(H(x)-H\left(x-\tfrac{2}{3}\ell\right)\right)-$$
$$-\sum_{k=0}^{\infty}\left\{H\left(t-x/a-2\left(k+\tfrac{1}{3}\right)\ell/a\right)-H\left(t+x/a-2\left(k+\tfrac{4}{3}\right)\ell/a\right)\right\}$$
$$\cdot H\left(x-\tfrac{2}{3}\ell\right)+\sum_{k=0}^{\infty}\left\{H\left(t-x/a-2\left(k+\tfrac{2}{3}\right)\ell/a\right)-H\left(t+x/a-2\left(k+\tfrac{5}{3}\right)\ell/a\right)\right\}$$
$$\cdot H\left(x-\tfrac{2}{3}\ell\right)+\sum_{k=0}^{\infty}\left\{H\left(t+x/a-2\left(k+\tfrac{4}{3}\right)\ell/a\right)-H\left(t+x/a-2\left(k+\tfrac{5}{3}\right)\ell/a\right)\right\}$$
$$\cdot\left(H(x)-H\left(x-\tfrac{2}{3}\ell\right)\right)-$$
(4)
$$-\sum_{k=0}^{\infty}\left\{H\left(t+x/a-2\left(k+\tfrac{5}{3}\right)\ell/a\right)-H\left(t+x/a-2(k+2)\ell/a\right)\right\}$$
$$\cdot\left(H(x)-H\left(x-\tfrac{2}{3}\ell\right)\right)-$$
$$-\sum_{k=0}^{\infty}\left\{H\left(t-x/a-2(k+1)\ell/a\right)-H\left(t+x/a-2(k+2)\ell/a\right)\right\}$$
$$\cdot H\left(x-\tfrac{2}{3}\ell\right)+$$
$$+\sum_{k=0}^{\infty}\left\{H\left(t-x/a-2\left(k+\tfrac{2}{3}\right)\ell/a\right)-H\left(t+x/a-2(k+1)\ell/a\right)\right\}$$
$$\cdot\left(H(x)-H\left(x-\tfrac{1}{3}\ell\right)\right)-$$
$$-\sum_{k=0}^{\infty}\left\{H\left(t-x/a-2(k+1)\ell/a\right)-H\left(t+x/a-2\left(k+\tfrac{4}{3}\right)\ell/a\right)\right\}\cdot\left(H(x)-H\left(x-\tfrac{1}{3}\ell\right)\right),$$

where a is the velocity of wave propagation in the rod.

Fig. 2 provides the signs of velocities in areas restricted by wave fronts.

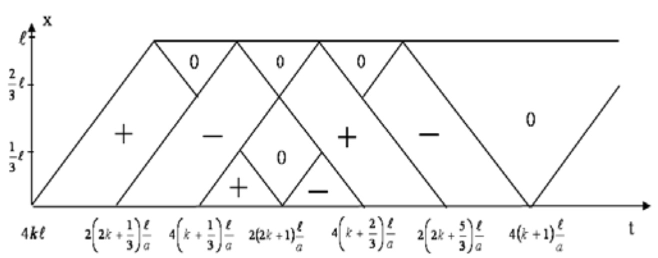

Fig. 2. *Signs of velocities in characteristic areas of motion.*

Substitution of (4) into system (3) reduces the initial nonlinear problem to a linear equation

(5) $$E\frac{\partial^2 U}{\partial x^2}=\rho\frac{\partial^2 U}{\partial t^2}+q\cdot\aleph(x,t)$$

where *U(x, t)* is the displacement of sections of the rod.

We can use the Laplace transform to solve this equation. After transformation, taking zero initial conditions into account, equation (2) and the boundary conditions assume the following form:

(6) $$\frac{d^2\overline{U}}{dx^2}-\frac{p^2}{a^2}\overline{U}-\frac{q}{E}\cdot\overline{\aleph}(p,x)=0\,,$$

(7) $$\overline{U}=0\quad on\quad x=\ell\,,$$

(8) $$\frac{d\overline{U}}{dx}=\frac{\sigma_0}{E}\left\{1-2\sum_{k=0}^{\infty}(-1)^k e^{-p\frac{2}{3}k\ell/a}\right\}\quad on\ \ x=0,$$

where p is transformation parameter.

Solution of the problem (6) - (8) in images has the following form:

$$\overline{U}(p,x)=\frac{a\sigma_0}{pE}\frac{1}{1+e^{-2p\ell/a}}\left\{1-2\sum_{k=0}^{\infty}(-1)^k e^{-p\frac{2}{3}k\ell/a}\right\}\times$$
$$\times\left(e^{p(x-2\ell)/a}-e^{-px/a}\right)-$$
$$-\frac{aq}{2pE}\frac{1}{1+e^{-2p\ell/a}}\left[\frac{a}{2p}H(x)\sum_{k=0}^{\infty}e^{p(x-2k\ell)/a}-\right.$$
$$-\left(\frac{a}{2p}+\frac{5}{3}\ell\right)\sum_{k=0}^{\infty}e^{p(x-2(k+1)\ell)/a}-\frac{a}{p}H(x)\sum_{k=0}^{\infty}e^{p\left(x-2\left(k+\frac{1}{3}\right)\ell\right)/a}+$$
$$+3\ell\sum_{k=0}^{\infty}e^{p\left(x-2\left(k+\frac{4}{3}\right)\ell\right)/a}-3\ell\sum_{k=0}^{\infty}e^{p\left(x-2\left(k+\frac{5}{3}\right)\ell\right)/a}+$$
$$+\left(\frac{5}{3}\ell-\frac{a}{2p}\right)H(x)\sum_{k=0}^{\infty}e^{p(x-2(k+2)\ell)/a}+\frac{a}{p}\sum_{k=0}^{\infty}e^{p\left(x-2\left(k+\frac{2}{3}\right)\ell\right)/a}-$$
$$-\frac{a}{p}\sum_{k=0}^{\infty}e^{p\left(x-2\left(k+\frac{8}{3}\right)\ell\right)/a}++\frac{a}{2p}\sum_{k=0}^{\infty}e^{p(x-2(k+3)\ell)/a}+$$
$$\left.+\frac{a}{p}\sum_{k=0}^{\infty}e^{p\left(x-2\left(k+\frac{7}{3}\right)\ell\right)/a}\right]+\frac{1}{1+e^{-2p\ell/a}}\frac{aq}{2pE}\left[\frac{a}{2p}\sum_{k=0}^{\infty}e^{-p(x+2k\ell)/a}-\right.$$
$$-\frac{a}{p}\sum_{k=0}^{\infty}e^{-p\left(x+2\left(k+\frac{1}{3}\right)\ell\right)/a}-\left(\frac{a}{2p}+\frac{5}{3}\ell\right)\sum_{k=0}^{\infty}e^{-p(x+2(k+1)\ell)/a}+$$
$$+3\ell\sum_{k=0}^{\infty}e^{-p\left(x+2\left(k+\frac{4}{3}\right)\ell\right)/a}--3\ell\sum_{k=0}^{\infty}e^{-p\left(x+2\left(k+\frac{5}{3}\right)\ell\right)/a}+$$
$$+\left(\frac{5}{3}\ell-\frac{a}{2p}\right)\sum_{k=0}^{\infty}e^{-p(x+2(k+2)\ell)/a}+\frac{a}{p}\sum_{k=0}^{\infty}e^{-p\left(x+2\left(k+\frac{2}{3}\right)\ell\right)/a}-$$
$$-\frac{a}{p}\sum_{k=0}^{\infty}e^{-p\left(x+2\left(k+\frac{8}{3}\right)\ell\right)/a}++\frac{a}{2p}\sum_{k=0}^{\infty}e^{-p(x+2(k+3)\ell)/a}+$$
$$\left.+\frac{a}{p}\sum_{k=0}^{\infty}e^{-p\left(x+2\left(k+\frac{7}{3}\right)\ell\right)/a}\right]++\frac{aq}{2pE}\left[\frac{a}{2p}H(x)\sum_{k=0}^{\infty}e^{p(x-2k\ell)/a}+\right.$$
$$+\frac{a}{p}H(x)\sum_{k=0}^{\infty}e^{p\left(x-2\left(k+\frac{2}{3}\right)\ell\right)/a}-H(x)\sum_{k=0}^{\infty}e^{-p(x+2k\ell)/a}\left(\frac{a}{2p}+x\right)+$$
(9)
$$+\sum_{k=0}^{\infty}e^{-p\left(x+2\left(k+\frac{1}{3}\right)\ell\right)/a}\times$$
$$\times\left\{H(x)\left(\frac{a}{p}+2x\right)-H\left(x-\tfrac{2}{3}\ell\right)\left(\frac{a}{p}+\left(x-\frac{2}{3}\ell\right)\right)\right\}-$$
$$-\frac{a}{p}H(x)\sum_{k=0}^{\infty}e^{-p\left(x+2\left(k+\frac{5}{3}\right)\ell\right)/a}+\sum_{k=0}^{\infty}e^{-p(x+2(k+1)\ell)/a}\times$$
$$\times\left\{xH(x)+H\left(x-\tfrac{2}{3}\ell\right)\left(\frac{a}{p}+\left(x-\frac{2}{3}\ell\right)\right)-\right.$$
$$\left.-H\left(x-\tfrac{1}{3}\ell\right)\left(\frac{a}{p}+\left(x-\frac{1}{3}\ell\right)\right)\right\}+\frac{a}{2p}H(x)\sum_{k=0}^{\infty}e^{-p(x+2(k+2)\ell)/a}+$$
$$+\frac{a}{p}H(x)\sum_{k=0}^{\infty}e^{-p\left(x+2\left(k+\frac{4}{3}\right)\ell\right)/a}+$$
$$+\sum_{k=0}^{\infty}e^{p(x-2(k+1)\ell)/a}\left\{-xH(x)+H\left(x-\tfrac{2}{3}\ell\right)\left(\frac{a}{p}-\left(x-\frac{2}{3}\ell\right)\right)-\right.$$

$$-H\left(x-\tfrac{1}{3}\ell\right)\left(\frac{a}{p}-\left(x-\frac{1}{3}\ell\right)\right)\Big\}\Big\}-\frac{a}{p}H(x)\sum_{k=0}^{\infty}e^{p\left(x-2\left(k+\frac{1}{3}\right)\ell\right)/a}+$$

$$+\sum_{k=0}^{\infty}e^{p\left(x-2\left(k+\frac{4}{3}\right)\ell\right)/a}\times$$

$$\times\left\{-H(x)\left(\frac{a}{p}-2x\right)+H\left(x-\tfrac{1}{3}\ell\right)\left(\frac{a}{p}-\left(x-\frac{1}{3}\ell\right)\right)\right\}+$$

$$+\sum_{k=0}^{\infty}e^{p\left(x-2\left(k+\frac{5}{3}\right)\ell\right)/a}\times$$

$$\times\left\{H(x)\left(\frac{a}{p}-2x\right)-H\left(x-\tfrac{2}{3}\ell\right)\left(\frac{a}{p}-\left(x-\frac{2}{3}\ell\right)\right)\right\}-$$

$$-H(x)\sum_{k=0}^{\infty}e^{p(x-2(k+2)\ell)/a}\times\times\left(\frac{a}{2p}-x\right)-\sum_{k=0}^{\infty}e^{-p\left(x+2\left(k+\frac{2}{3}\right)\ell\right)/a}\times$$

$$\times\left\{H(x)\left(\frac{a}{p}+2x\right)-H\left(x-\tfrac{1}{3}\ell\right)\left(\frac{a}{p}+\left(x-\frac{1}{3}\ell\right)\right)\right\}\Big].$$

This bulky generalized solution of the problem acquires very compact expressions if they are recorded in appropriate characteristic areas. On the basis of (9), solutions for a number of the first specific areas – in the case of this problem, for 51 areas – were recorded, and then, using the method of mathematical induction, solutions for all areas covering semi-infinite strips $0\le x\le\ell,\ t>0$ were recorded.

Investigation of solutions allows us to distinguish 17 areas that characterize one full oscillation of the system. Solutions of problems on the areas that characterize the two subsequent full oscillations were obtained. They are omitted for the sake of brevity.

Solution of the problem at $0<t<\infty$ is written in subsequent specific areas with any numbers. In general view, let us represent them correspondingly as $17k+1,\ 17k+2,\ \ldots,17(k+1)$, $(k=0,1,2,\ldots)$ (Fig. 3).

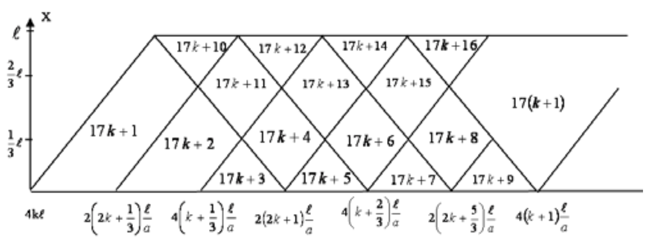

***Fig. 3.** Characteristic areas of the motion plane.*

Then, for the specific area $17k+1$ bounded by characteristics:

$$at<2(2k+1)\ell-x\ ,\quad x<at<2\left(\frac{1}{3}\ell+2k\right)+x,\qquad 0\le x\le\ell$$

stresses and velocities will have the following expressions:

$$\sigma_{17k+1}=-\sigma_0+\frac{q}{2}x,\qquad \vartheta_{17k+1}=\vartheta_0-\frac{aq}{2E}(at-4k\ell).$$

In areas $17k+2$:

$$x+2\left(2k+\frac{1}{3}\ell\right)<at<x+4\left(4k+\frac{1}{3}\right)\ell,\ at<2(2k+1)\ell\ ,\qquad 0\le x\le\frac{2}{3}\ell$$

solutions are

$$\sigma_{17k+2}=\sigma_0-\frac{q}{2}x,\qquad \vartheta_{17k+2}=-\vartheta_0+\frac{aq}{2E}\left(at-4\left(k+\frac{1}{3}\right)\ell\right)$$

In triangular areas $17k+3$:

$$x+4\left(k+\frac{1}{3}\right)\ell<at<2(2k+1)\ell-x\ ,\qquad 0\le x\le\frac{1}{3}\ell$$

solutions have the following expressions

$$\sigma_{17k+3}=-\sigma_0+\frac{q}{2}x\ ,\qquad \vartheta_{17k+3}=\vartheta_0-\frac{aq}{2E}\left(at-\left(4k+\frac{1}{3}\right)\ell\right).$$

In areas $17k+4$

$$x+4\left(k+\frac{1}{3}\right)\ell<at<x+2(2k+1)\ell,$$

$$2(2k+1)\ell-x<at<4\left(k+\frac{2}{3}\right)\ell-x$$

we have

$$\sigma_{17k+4}=-2\sigma_0+\frac{1}{3}ql\ ,\qquad \vartheta_{17k+4}=0\ .$$

In specific areas $17k+5$,

$$x+2(2k+1)\ell<at<4\left(k+\frac{2}{3}\right)\ell-x\ ,\qquad 0\le x\le\frac{1}{3}\ell,$$

the unknown functions will be

$$\sigma_{17k+5}=\sigma_0-\frac{q}{2}x\ ,\qquad \vartheta_{17k+5}=-\vartheta_0+\frac{aq}{2E}\left(at-\left(4k+\frac{2}{3}\right)\ell\right).$$

Consideration of the solution in the areas $17k+6$:

$$x+2(2k+1)\ell<at<x+4\left(k+\frac{2}{3}\right)\ell,$$

$$4\left(k+\frac{2}{3}\right)\ell-x<at<2\left(2k+\frac{5}{3}\right)\ell-x$$

gives

$$\sigma_{17k+6}=\sigma_0+\frac{q}{2}(x-3\ell),\quad \vartheta_{17k+6}=\vartheta_0-\frac{aq}{2E}(at-(4k+1)\ell)\ .$$

In areas $17k+7$:

$$x+4\left(k+\frac{2}{3}\right)\ell<at<2\left(2k+\frac{5}{3}\right)\ell-x\ ,\qquad 0\le x\le\frac{1}{3}\ell,$$

the solution is defined by the following expressions:

$$\sigma_{17k+7}=-\sigma_0+\frac{q}{2}x\ ,\qquad \vartheta_{17k+7}=\vartheta_0-\frac{aq}{2E}(at-(4k-1)\ell).$$

In areas $17k+8$,

$$x+4\left(k+\frac{2}{3}\right)\ell<at<x+2\left(2k+\frac{5}{3}\right)\ell,$$

$$2\left(2k+\frac{5}{3}\right)\ell-x<at<4(k+1)\ell-x$$

solution is

$$\sigma_{17k+8}=-\sigma_0-\frac{q}{2}(x-3\ell),\quad \vartheta_{17k+8}=-\vartheta_0+\frac{aq}{2E}\left(at-\left(4k+\frac{17}{3}\right)\ell\right).$$

In rectangular areas $17k+9$:

$$x+2\left(2k+\frac{5}{3}\right)\ell<at<4(k+1)\ell-x\ ,\qquad 0\le x\le\frac{1}{3}\ell,$$

we obtain

$$\sigma_{17k+9} = \sigma_0 - \frac{q}{2}x, \qquad \vartheta_{17k+9} = -\vartheta_0 + \frac{aq}{2E}\left(at - 2\left(2k + \frac{2}{3}\right)\ell\right).$$

In areas $17k+10$

$$2(2k+1)\ell - x < at < x + 2\left(2k + \frac{1}{3}\right)\ell, \qquad \frac{2}{3}\ell \le x \le \ell$$

we obtain

$$\sigma_{17k+10} = -2\sigma_0 + ql, \qquad \vartheta_{17k+10} = 0.$$

In triangular areas $17k+11$:

$$x + 2\left(2k + \frac{1}{3}\right)\ell < at < x + 4\left(k + \frac{1}{3}\right)\ell,$$

$$2(2k+1)\ell - x < at < 4\left(k + \frac{2}{3}\right)\ell - x$$

solution will be

$$\sigma_{17k+11} = -\frac{q}{2}\left(x - \frac{4}{3}\ell\right), \qquad \vartheta_{17k+11} = -2\vartheta_0 + \frac{aq}{2E}(at - 4k\ell).$$

Analysis of the solution in areas $17k+12$:

$$x + 4\left(k + \frac{1}{3}\right)\ell < at < 4\left(k + \frac{2}{3}\right)\ell - x, \qquad \frac{2}{3}\ell \le x \le \ell$$

gives

$$\sigma_{17k+12} = 2\sigma_0 - \frac{2}{3}ql, \qquad \vartheta_{17k+12} = 0.$$

In specific areas $17k+13$:

$$x + 4\left(k + \frac{1}{3}\right)\ell < at < x + 2(2k+1)\ell,$$

$$4\left(k + \frac{2}{3}\right)\ell - x < at < 2\left(2k + \frac{5}{3}\right)\ell - x$$

the unknown functions will be

$$\sigma_{17k+13} = \frac{q}{2}\left(x - \frac{4}{3}\ell\right), \qquad \vartheta_{17k+13} = 2\vartheta_0 - \frac{aq}{2E}\left(at - \left(4k + \frac{1}{2}\right)\ell\right).$$

In areas $17k+14$:

$$2\left(2k + \frac{5}{3}\right)\ell - x < at < x + 2(2k+1)\ell, \qquad \frac{2}{3}\ell \le x \le \ell$$

we obtain

$$\sigma_{17k+14} = -2\sigma_0 + \frac{q}{2}l, \qquad \vartheta_{17k+14} = 0.$$

In the triangular areas $17k+15$:

$$x + 2(2k+1)\ell < at < x + 4\left(k + \frac{2}{3}\right)\ell, \qquad 2\left(2k + \frac{5}{3}\right)\ell - x < at < 4(k+1)\ell - x$$

we have

$$\sigma_{17k+15} = \sigma_0 - \frac{q}{2}\left(x - \frac{2}{3}\ell\right), \quad \vartheta_{17k+15} = -\vartheta_0 + \frac{aq}{2E}\left(at - \left(4k + \frac{5}{3}\right)\ell\right).$$

In areas $17k+16$:

$$4(k+1)\ell - x < at < x + 4\left(k + \frac{2}{3}\right)\ell, \qquad \frac{2}{3}\ell \le x \le \ell$$

solutions have the following form:

$$\sigma_{17k+16} = -\frac{3}{2}q\ell, \qquad \vartheta_{17k+16} = 0.$$

Further analysis of the problem shows that in the trapezoidal areas $17(k+1)$:

$$at > 4(k+1)\ell - x, \qquad 0 \le x \le \frac{2}{3}\ell$$

$$at > x + 4\left(k + \frac{2}{3}\right)\ell, \qquad \frac{2}{3}\ell < x \le \ell$$

$$at < x + 4(k+1)\ell, \qquad 0 \le x \le \ell$$

the system turns out to be at rest:

$$\sigma_{17(k+1)} = 0, \qquad \vartheta_{17(k+1)} = 0.$$

Fig. 4 provides the oscillation curve of the end point of the rod.

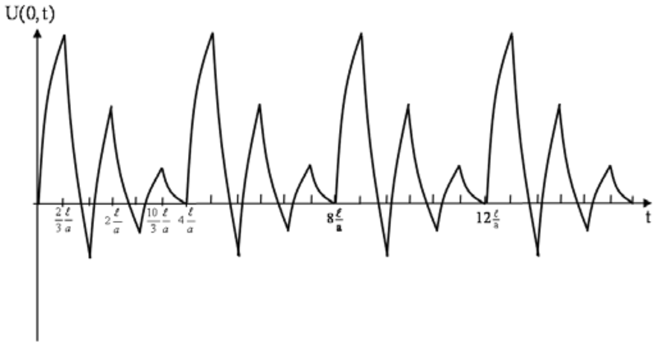

Fig. 4. *Oscillation curve of the end point of the rod.*

The resulting solution of the nonlinear problem of wave propagation in the system under consideration under cyclic step load (1) with a frequency three times greater than the rod's free frequencies indicate that the construction with dry friction under investigation shows steady periodic oscillations with a period $4\ell/a$.

3. Conclusion

Based on the results of the solutions obtained, the following conclusion can be drawn: there is a class of cyclic loads with a frequency that is an arbitrary integer times greater than the frequency of the oscillations of the system, under the action of which the system will have steady– subharmonic oscillations with two frequencies. Besides, one oscillation coincides with the frequency of the oscillation of the system, the other - with the frequency of external loading. For each subsequent time range equal to *n* periods of external loading, the system begins oscillations from the rest and undisturbed state under the load– identical to the load at $0 < t < 4\ell/a$.

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