MOTION, START AND BRAKING OF RAILWAY TRAIN

Doctor of Physical and Mathematics sciences, Professor, Member of the Engineering Academy of the RoK Tyurekhodjayev A.N. [1],
Doctor of Physical and Mathematics sciences, Professor Ibrayev A.G.[2]
Kazakh National Technical University named after K.I. Satpayev, Kazakhstan, Almaty,
Eurasian National University named after L.N. Gumilev, Kazakhstan, Astana,

Abstract: *The analytical solution of the problem of longitudinal deformation of rail with taking into account dry friction on the "wheel-rail" contact, rails junctions, number of axles of the railway train from the position of the wave theory is provided.*
Determination of the rail dynamic stress-strain state while driving a railway train on it and taking into account the number of parameters which characterizing the movement is the actual problem, which has a great importance for practical operation of rail transport. In the mathematical modeling of railway transport and determining the dynamic stress-strain rail the not significant parameters is usually neglected. There are other parameters that need to take into account. For example, in the problem of deformation and rail tension can not be neglected dry friction in the "wheel-rail" contact. These settings include the dry friction in the "wheel-rail" contact. In the case of neglecting of friction system of forces applied to the rail, is balanced and the only movement that is possible is rotation of the wheel around its center of mass. If the friction is neglected the law on the conservation of momentum in classical mechanics is being broken. Thus, neglecting of rolling friction is not allowed.
Numerous reflections of the waves from the wheels and the rail ends cannot be neglected. In this paper we first made records of friction, impact of ties, wave reflection and contribution of junctions to the dynamic stress-strain state of the rail.
KEYWORDS: *RAILWAY TRANSPORT, DRY FRICTION, «WHEEL-RAIL» CONTACT, RAIL JUNCTIONS, MATHEMATICAL MODEL, MOTION DYNAMICS, WAVE THEORY*

1. Introduction

Determination of the dynamic stress-strain state of the rail while movement of a train on it and taking into account the huge number of parameters that characterize the movement is the actual problem, which has great importance for the practice of exploitation of rail transport.

In this paper, a record of the elastic deformation of the rail, number of rails, dry friction in the «wheel-rail» contact, their junctions, sleepers' location intensity, wheels shocks on the ends of the rails, distribution and reflection of waves from the wheels and the ends of the rails, amount of wagons axes, starting and braking is done.

Neglecting friction at the «wheel-rail» contact, as it is done, for example in S. P. Timoshenko's [1,2] and other's papers it leads to a violation of the law of conservation of momentum and the train is stationary. And, if the train is still moving, it is just because friction, which is discarded in theory, is present in practice, providing indispensable movement of the train. In this paper the starting and braking is considered, carrying a variable rate.

2. Prerequisites and means for solving the problem

At the movement of railway train, which consists of four-axis cars, with acceleration a_τ, dynamic load interaction of wheels with rails is written as follows:

$$P(x,t) = -\frac{\tau_k}{EF}\sum_{r=0}^{s}\Big\{\delta\big[a_\tau t^2 + v_0 t - x - r(2l_1 + l_2 + l_3)\big] + \delta\big[a_\tau t^2 + v_0 t - x - (2r+1)l_1 - r(l_2 + l_3)\big] + \delta\big[a_\tau t^2 + v_0 t - x - (2r+1)l_1 - (r+1)l_2 - rl_3\big] + \delta\big[a_\tau t^2 + v_0 t - x - 2(r+1)l_1 - (r+1)l_2 - rl_3\big]\Big\} \quad (1)$$

where τ_k – the value associated with contact with dry friction bearings, l_1, l_2, l_3 – the distance between wheels, *s* – the number of cars, *E*– the elastic modulus, *F*– cross-sectional area of the rail, $\delta(z)$ – Dirac delta function, v_0 – initial speed.

The differential equation of motion, taking into account (1) is described as follows [3]:

$$\frac{\partial^2 u(x,t)}{\partial x^2} - \frac{1}{a^2}\cdot\frac{\partial^2 u(x,t)}{\partial t^2} - \alpha\cdot u(x,t) = -\frac{\tau_k}{EF}\sum_{r=0}^{s}\Big\{\delta\big[a_\tau t^2 + v_0 t - x - r(2l_1 + l_2 + l_3)\big] + \delta\big[a_\tau t^2 + v_0 t - x - (2r+1)l_1 - r(l_2 + l_3)\big] + \delta\big[a_\tau t^2 + v_0 t - x - (2r+1)l_1 - (r+1)l_2 - rl_3\big] + \delta\big[a_\tau t^2 + v_0 t - x - 2(r+1)l_1 - (r+1)l_2 - rl_3\big]\Big\} \quad (2)$$

where α - coefficient of stiffness of the elastic connection, $u(x,t)$ - shift of the rail section.

The initial and boundary conditions are

$$t = 0,\ u(x,0) = 0,\ \frac{\partial u(x,0)}{\partial t} = 0 \quad (3)$$

$$x = 0,\ \sigma(0,t) = E\frac{\partial u(0,t)}{\partial x} = -\sigma_0\sum_{r=0}^{s}\left[\delta\left(t - \frac{r(2l_1 + l_2 + l_3)}{v_0}\right) + \delta\left(t - \frac{(2r+1)l_1 + r(l_2 + l_3)}{v_0}\right) + \delta\left(t - \frac{(2r+1)l_1 + (r+1)l_2 + rl_3}{v_0}\right) + \delta\left(t - \frac{2(r+1)l_1 + (r+1)l_2 + rl_3}{v_0}\right)\right] \quad (4)$$

at

$$x = L,\quad \sigma(L,t) = E\frac{\partial u(L,t)}{\partial x} = 0 \quad (5)$$

Equation (2), taking into account the properties of generalized functions [4,5], is shown as the following form:

$$\frac{\partial^2 u(x,t)}{\partial x^2} - \frac{1}{a^2}\cdot\frac{\partial^2 u(x,t)}{\partial t^2} - \alpha\cdot u(x,t) = -\frac{\tau_k}{2EF}\sum_{r=0}^{s}\left\{\frac{1}{\sqrt{T_1^2 + T_x\left[x + 2l_1 + l_2 + l_3\right]}}\times\right. \quad (6)$$

$$\times\left[\delta\left(t-\sqrt{T_1^2+T_x\left[x+2l_1+l_2+l_3\right]}+T_1\right)+\right.$$
$$\left.+\delta\left(t+\sqrt{T_1^2+T_x\left[x+2l_1+l_2+l_3\right]}+T_1\right)\right]+$$
$$+\frac{1}{\sqrt{T_1^2+T_x\left[x+(2r+1)l_1+r(l_2+l_3)\right]}}\times$$
$$\times\left[\delta\left(t-\sqrt{T_1^2+T_x\left[x+(2r+1)l_1+r(l_2+l_3)\right]}+T_1\right)+\right.$$
$$\left.+\delta\left(t+\sqrt{T_1^2+T_x\left[x+(2r+1)l_1+r(l_2+l_3)\right]}+T_1\right)\right]+$$
$$+\frac{1}{\sqrt{T_1^2+T_x\left[x+(2r+1)l_1+(r+1)l_2+rl_3\right]}}\times$$
$$\times\left[\delta\left(t-\sqrt{T_1^2+T_x\left[x+(2r+1)l_1+(r+1)l_2+rl_3\right]}+T_1\right)+\right.$$
$$\left.+\delta\left(t+\sqrt{T_1^2+T_x\left[x+(2r+1)l_1+(r+1)l_2+rl_3\right]}+T_1\right)\right]+$$
$$+\frac{1}{\sqrt{T_1^2+T_x\left[x+2(r+1)l_1+(r+1)l_2+rl_3\right]}}\times$$
$$\times\left[\delta\left(t-\sqrt{T_1^2+T_x\left[x+2(r+1)l_1+(r+1)l_2+rl_3\right]}+T_1\right)+\right.$$
$$\left.\left.+\delta\left(t+\sqrt{T_1^2+T_x\left[x+2(r+1)l_1+(r+1)l_2+rl_3\right]}+T_1\right)\right]\right\}$$

where

$$T_1=\frac{v_0}{2a_\tau};\qquad T_x=\frac{1}{a_\tau}\cdot$$

3. Results and discussion

Using the Laplace-Carson integral and partial sampling method [2], we get a solution in the form of:

(7)
$$u(x,t)=-\frac{\tau_k\cdot a}{8EF}\sum_{k=1}^{m}(x_k+x_{k+1})\sum_{n=0}^{\infty}\sum_{r=0}^{n}(-1)^{n+r}\left(\frac{b}{2}\right)^{2n}\frac{t^{2r+1}}{(n-r)!r!(2r+1)}\times$$
$$\times\left\{\left(\frac{x-x_k}{a}\right)^{2(n-r)}\times\right.$$
$$\times\left[H\left(t+T_1-\sqrt{T_1^2+T_x\cdot\left[x_k+2l_1+l_2+l_3\right]}\right)+\right.$$
$$\left.+H\left(t+T_1+\sqrt{T_1^2+T_x\cdot\left[x_k+2l_1+l_2+l_3\right]}\right)\right]-$$
$$-\left(\frac{x-x_{k+1}}{a}\right)^{2(n-r)}\times$$
$$\times H\left(t+T_1-\sqrt{T_1^2+T_x\cdot\left[x_{k+1}+2l_1+l_2+l_3\right]}\right)+$$
$$+H\left(t+T_1+\sqrt{T_1^2+T_x\cdot\left[x_{k+1}+2l_1+l_2+l_3\right]}\right)+$$
$$+\left(\frac{x-x_k}{a}\right)^{2(n-r)}\times$$
$$\times\left[H\left(t+T_1-\sqrt{T_1^2+T_x\cdot\left[x_k+(2r+1)l_1+r(l_2+l_3)\right]}\right)+\right.$$
$$\left.+H\left(t+T_1+\sqrt{T_1^2+T_x\cdot\left[x_k+(2r+1)l_1+r(l_2+l_3)\right]}\right)\right]-$$
$$-\left(\frac{x-x_{k+1}}{a}\right)^{2(n-r)}\times$$
$$\times\left[H\left(t+T_1-\sqrt{T_1^2+T_x\cdot\left[x_{k+1}+(2r+1)l_1+r(l_2+l_3)\right]}\right)+\right.$$
$$\left.+H\left(t+T_1+\sqrt{T_1^2+T_x\cdot\left[x_{k+1}+(2r+1)l_1+r(l_2+l_3)\right]}\right)\right]-$$
$$+\left(\frac{x-x_k}{a}\right)^{2(n-r)}\times$$
$$\times\left[H\left(t+T_1-\sqrt{T_1^2+T_x\cdot\left[x_k+(2r+1)l_1+(r+1)l_2+rl_3\right]}\right)+\right.$$
$$\left.+H\left(t+T_1+\sqrt{T_1^2+T_x\cdot\left[x_k+(2r+1)l_1+(r+1)l_2+rl_3\right]}\right)\right]-$$
$$-\left(\frac{x-x_{k+1}}{a}\right)^{2(n-r)}\times$$
$$\times\left[H\left(t+T_1-\sqrt{T_1^2+T_x\cdot\left[x_{k+1}+(2r+1)l_1+(r+1)l_2+rl_3\right]}\right)+\right.$$
$$\left.+H\left(t+T_1+\sqrt{T_1^2+T_x\cdot\left[x_{k+1}+(2r+1)l_1+(r+1)l_2+rl_3\right]}\right)\right]-$$
$$+\left(\frac{x-x_k}{a}\right)^{2(n-r)}\times$$
$$\times\left[H\left(t+T_1-\sqrt{T_1^2+T_x\cdot\left[x_k+2(r+1)l_1+(r+1)l_2+rl_3\right]}\right)+\right.$$
$$\left.+H\left(t+T_1+\sqrt{T_1^2+T_x\cdot\left[x_k+2(r+1)l_1+(r+1)l_2+rl_3\right]}\right)\right]-$$
$$-\left(\frac{x-x_{k+1}}{a}\right)^{2(n-r)}\times$$
$$\times\left[H\left(t+T_1-\sqrt{T_1^2+T_x\cdot\left[x_{k+1}+2(r+1)l_1+(r+1)l_2+rl_3\right]}\right)+\right.$$
$$\left.\left.+H\left(t+T_1+\sqrt{T_1^2+T_x\cdot\left[x_{k+1}+2(r+1)l_1+(r+1)l_2+rl_3\right]}\right)\right]\right\}+$$
$$+\frac{\sigma_0\cdot a}{E}\cdot J_0\left(b\sqrt{t^2-\left(\frac{x}{a}\right)^2}\right)H\left(t-\frac{x}{a}\right)+$$
$$+\frac{\tau_k\cdot a}{8EF}\sum_{k=1}^{m}(x_k+x_{k+1})\sum_{n=0}^{\infty}\sum_{r=0}^{n}(-1)^{n+r}\left(\frac{b}{2}\right)^{2n}\frac{t^{2r+1}}{(n-r)!r!(2r+1)}\times$$
$$\times\left\{\left(\frac{x_k-x}{a}\right)^{2(n-r)}\times\right.$$
$$\times\left[H\left(t+T_1-\sqrt{T_1^2+T_x\cdot\left[x_k+2l_1+l_2+l_3\right]}\right)+\right.$$
$$\left.+H\left(t+T_1+\sqrt{T_1^2+T_x\cdot\left[x_k+2l_1+l_2+l_3\right]}\right)\right]-$$
$$-\left(\frac{x_{k+1}-x}{a}\right)^{2(n-r)}\times$$
$$\times\left[H\left(t+T_1-\sqrt{T_1^2+T_x\cdot\left[x_{k+1}+2l_1+l_2+l_3\right]}\right)+\right.$$
$$\left.+H\left(t+T_1+\sqrt{T_1^2+T_x\cdot\left[x_{k+1}+2l_1+l_2+l_3\right]}\right)\right]+$$
$$+\left(\frac{x_k-x}{a}\right)^{2(n-r)}\times$$
$$\times\left[H\left(t+T_1-\sqrt{T_1^2+T_x\cdot\left[x_k+(2r+1)l_1+r(l_2+l_3)\right]}\right)+\right.$$
$$\left.+H\left(t+T_1+\sqrt{T_1^2+T_x\cdot\left[x_k+(2r+1)l_1+r(l_2+l_3)\right]}\right)\right]-$$
$$-\left(\frac{x_{k+1}-x}{a}\right)^{2(n-r)}\times$$
$$\times\left[H\left(t+T_1-\sqrt{T_1^2+T_x\cdot\left[x_{k+1}+(2r+1)l_1+r(l_2+l_3)\right]}\right)+\right.$$
$$\left.+H\left(t+T_1+\sqrt{T_1^2+T_x\cdot\left[x_{k+1}+(2r+1)l_1+r(l_2+l_3)\right]}\right)\right]-$$
$$+\left(\frac{x_k-x}{a}\right)^{2(n-r)}\times$$
$$\times\left[H\left(t+T_1-\sqrt{T_1^2+T_x\cdot\left[x_k+(2r+1)l_1+(r+1)l_2+rl_3\right]}\right)+\right.$$
$$\left.+H\left(t+T_1+\sqrt{T_1^2+T_x\cdot\left[x_k+(2r+1)l_1+(r+1)l_2+rl_3\right]}\right)\right]-$$
$$-\left(\frac{x-x_{k+1}}{a}\right)^{2(n-r)}\times$$
$$\times\left[H\left(t+T_1-\sqrt{T_1^2+T_x\cdot\left[x_{k+1}+(2r+1)l_1+(r+1)l_2+rl_3\right]}\right)+\right.$$
$$\left.+H\left(t+T_1+\sqrt{T_1^2+T_x\cdot\left[x_{k+1}+(2r+1)l_1+(r+1)l_2+rl_3\right]}\right)\right]-$$
$$+\left(\frac{x_k-x}{a}\right)^{2(n-r)}\times$$
$$\times\left[H\left(t+T_1-\sqrt{T_1^2+T_x\cdot\left[x_k+2(r+1)l_1+(r+1)l_2+rl_3\right]}\right)+\right.$$
$$\left.+H\left(t+T_1+\sqrt{T_1^2+T_x\cdot\left[x_k+2(r+1)l_1+(r+1)l_2+rl_3\right]}\right)\right]-$$
$$-\left(\frac{x_{k+1}-x}{a}\right)^{2(n-r)}\times$$
$$\times\left[H\left(t+T_1-\sqrt{T_1^2+T_x\cdot\left[x_{k+1}+2(r+1)l_1+(r+1)l_2+rl_3\right]}\right)+\right.$$
$$\left.\left.+H\left(t+T_1+\sqrt{T_1^2+T_x\cdot\left[x_{k+1}+2(r+1)l_1+(r+1)l_2+rl_3\right]}\right)\right]\right\}+$$

$$
\begin{aligned}
&+\frac{\sigma_0\cdot a}{E}\cdot J_0\left(b\sqrt{t^2-\left(\frac{x-2L}{a}\right)^2}\right)H\left(t-\frac{x-2L}{a}\right)-\\
&-\frac{\tau_k}{4EF}\sum_{k=1}^{m}\left(x_k+x_{k+1}\right)\left\{\frac{1}{\sqrt{T_1^2+T_x\cdot\left[x+2l_1+l_2+l_3\right]}}\times\right.\\
&\times\Big\{\Big[\delta\Big(t+T_1-\sqrt{T_1^2+T_x\left[x_k+2l_1+l_2+l_3\right]}\Big)+\\
&+\delta\Big(t+T_1+\sqrt{T_1^2+T_x\left[x_k+2l_1+l_2+l_3\right]}\Big)\Big]\delta\left(x-x_k\right)-\\
&-\Big[\delta\Big(t+T_1-\sqrt{T_1^2+T_x\left[x_{k+1}+2l_1+l_2+l_3\right]}\Big)+\\
&+\delta\Big(t+T_1+\sqrt{T_1^2+T_x\left[x_{k+1}+2l_1+l_2+l_3\right]}\Big)\Big]\delta\left(x-x_{k+1}\right)\Big\}+\\
&+\frac{1}{\sqrt{T_1^2+T_x\left[x+(2r+1)l_1+r(l_2+l_3)\right]}}\times\\
&\times\Big\{\Big[\delta\Big(t+T_1-\sqrt{T_1^2+T_x\left[x_k+(2r+1)l_1+r(l_2+l_3)\right]}\Big)+\\
&+\delta\Big(t+T_1+\sqrt{T_1^2+T_x\left[x_k+(2r+1)l_1+r(l_2+l_3)\right]}\Big)\Big]\delta\left(x-x_k\right)-\\
&-\Big[\delta\Big(t+T_1-\sqrt{T_1^2+T_x\left[x_{k+1}+(2r+1)l_1+r(l_2+l_3)\right]}\Big)+\\
&+\delta\Big(t+T_1+\sqrt{T_1^2+T_x\left[x_{k+1}+(2r+1)l_1+r(l_2+l_3)\right]}\Big)\Big]\delta\left(x-x_{k+1}\right)\Big\}+\\
&+\frac{1}{\sqrt{T_1^2+T_x\left[x+(2r+1)l_1+(r+1)l_2+rl_3\right]}}\times\\
&\times\Big\{\Big[\delta\Big(t+T_1-\sqrt{T_1^2+T_x\left[x_k+(2r+1)l_1+(r+1)l_2+rl_3\right]}\Big)+\\
&+\delta\Big(t+T_1+\sqrt{T_1^2+T_x\left[x_k+(2r+1)l_1+(r+1)l_2+rl_3\right]}\Big)\Big]\delta\left(x-x_k\right)-\\
&-\Big[\delta\Big(t+T_1-\sqrt{T_1^2+T_x\left[x_{k+1}+(2r+1)l_1+(r+1)l_2+rl_3\right]}\Big)+\\
&+\delta\Big(t+T_1+\sqrt{T_1^2+T_x\left[x_{k+1}+(2r+1)l_1+(r+1)l_2+rl_3\right]}\Big)\Big]\delta\left(x-x_{k+1}\right)\Big\}+\\
&+\frac{1}{\sqrt{T_1^2+T_x\left[x+2(r+1)l_1+(r+1)l_2+rl_3\right]}}\times\\
&\times\Big\{\Big[\delta\Big(t+T_1-\sqrt{T_1^2+T_x\left[x_k+2(r+1)l_1+(r+1)l_2+rl_3\right]}\Big)+\\
&+\delta\Big(t+T_1+\sqrt{T_1^2+T_x\left[x_k+2(r+1)l_1+(r+1)l_2+rl_3\right]}\Big)\Big]\delta\left(x-x_k\right)-\\
&-\Big[\delta\Big(t+T_1-\sqrt{T_1^2+T_x\left[x_{k+1}+2(r+1)l_1+(r+1)l_2+rl_3\right]}\Big)+\\
&+\delta\Big(t+T_1+\sqrt{T_1^2+T_x\left[x_{k+1}+2(r+1)l_1+(r+1)l_2+rl_3\right]}\Big)\Big]\delta\left(x-x_{k+1}\right)\Big\}\Big\}.
\end{aligned}
$$

4. Conclusion

In this paper the regularities are set in the relaxation change of stresses and velocities of motion.

Analytical solutions for the longitudinal and flexural deformation of the rail with all the other factors listed above are achieved.

5. References

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