THE TURBO FAN TRANSPORT AIRCRAFT DINAMIC COST INDEX DETERMINATION METHOD BY SPECIFIC RANGE FUNCTIONS

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***Abstract*:**
The air traffic network in European Union (EU) is a patchwork blanket of over 650 different sectors, which generate airspace capacity problem resulting in delays. With airspace as complex as this, aircraft cannot made flight operations, when it would be ideal to do so in operational terms, also this means that aircraft cannot fly at optimal cruising flight level. The generated delays increase airline direct operative costs and indirect operative costs as a result of unsatisfactory of passengers, lost of connecting flight, daily airline time table problems arise from delay of previous flights, etc. The passengers use air transport as a solution for they transport needs because air transport speed, punctuality and connectivity with other transport solutions. Delays generated on ground or in the air are lost of business time and therefore money for passengers and airline, which must compensate with minimal consequences. The vast number of study was published dealing with delay problem in air transport. One of recent idea was proposed by the University of Westminster and supported by Eurocontrol and concerning of implementation of Dynamic Cost Index (DCI). Unfortunately, the airline still does not have operative method for application of DCI. In the paper is presented method of operative determination of DCI according to delay value, under criterion of minimum fuel increase, due decreasing of flight time. As damage arise from implementation of DCI is increase of fuel consumption. The presented model is ready for operational use in airline industry. The paper present method of DCI determination for medium range routes or intra EU routes for aircraft B737800, but it can easily develop for long-haul flights. The flight data are determined from manufacturer's Flight Planning and Performance Manual. Proposed DCI determination method in the paper can be implemented in area of airline strategy, management and operations, Low Cost Carriers and airline competition and airline and airport performance;

Key words: DYNAMIC COST INDEX, AIRLINE, DELAY, COSTS

1. Introduction

A large number of flights around the world are deviate from the planned timetable. This deviation, called the delay, is one of the biggest problems they face every day as airlines and passengers. Delays, in addition to being an inconvenience for passengers who must wait in airport building, often for hours, can lead to that the same mistakes the next flight, late to business meetings and the like. On the other hand, the airline delays may adversely affect in a material and operational terms. Companies often have to pay compensation to passengers, or overwork their crews. The delay of one flight can lead to disorders of the whole rotation of an aircraft, but also affect the rotation of other aircraft in the fleet. In addition, frequent and long delays can and demean the reputation of the company. The issue of delay becomes even more interesting for the airline if you take into account the situation we are in today is a system of air traffic. Air traffic from year to year increases in size and demand for it is growing annually by about five percent. The largest airports already operating in saturation conditions, or is close to reaching its capacity, which extends, if at all possible, requires a lot of time and resources. On the other hand, many airlines have significant problems in functioning and lead the fight for financial survival in the global economic crisis and the very strong competition. Taking into account the facts presented, it is clear that airlines must seek ways to make it possible to reduce delays or more reduce the negative impacts of this delay. Taking into account the mentioned fact about the limited capacity and expandability of the same in major airports, it is clear that solutions to the problem of airline delays must be sought in better organization of their activities. This paper presents a way of fighting against the negative impact of delays that could apply to airlines and to the concept of *Dynamic Cost Index - DCI*, which aims at reducing the negative impact of delays to a minimum, i.e. minimize the costs incurred due to delays. Prior to the detailed description of the concept of *dynamic cost index* describes the concepts of cost index (*Cost Index - CI*) and delays.

2. The mathematical model

Taking into account the previous assumptions, the mathematical model used in this example can be formulated as a variation of the model presented in chapter 1, and now takes the form of the model mixed-integer programming, presented in the following way:

Minimization function or objective function:

$$F = \sum_{i=FL_{\min}}^{FL_{\max}} \sum_{j=CI_{\min}}^{CI_{\max}} x_{ij} (\Delta x_{Fij} C_{FK} + (K - \Delta x_{Tij}) C_{TKij}) \quad (14)$$

Under constraints:

$$\begin{aligned}
&\sum_i \sum_j x_{ij} = 1\\
&\Delta x_{Fij} = x_{F\,\mathrm{Re}f} - x_{Fij}, \forall (i,j)\\
&\Delta x_{Tij} = x_{T\,\mathrm{Re}f} - x_{Tij}, \forall (i,j)\\
&C_{TKij} = f(K - \Delta x_{Tij}), \forall (i,j)\\
&x_{ij} = \{0,1\}, \forall (i,j)\\
&(i,j)_{\mathrm{Re}f} = (FL370, CI20)\\
&i = \{FL290, FL310, FL330, FL350, FL370\}\\
&j = \{CI0, CI20, CI40, CI60, CI100\}.
\end{aligned} \quad (15)$$

In objective function and constraints we use following variables: F is denoted to goal function, x_{ij} is denoted to binary variable which corresponds to each combination of i and j, Δx_{Fij} is denoted to difference in fuel consumption compared to reference case, C_{FK} is denoted to fuel price (Euro/kg), K is denoted to delay in concern, Δx_{Tij} is denoted to difference in flight time compared to reference case, C_{TKij} is denoted to delay costs for every combination of i and j (Euro/min), x_{FRef} and x_{TRef} is denoted to fuel consumption value and flight time in reference case respectively, x_{Fij} and x_{Tij} is denoted to flight fuel consumption and flight time for every combination i and j (Table 2.),$(i, j)_{Ref}$ is denoted to reference case.As can be seen, in contrast to the model (11) (12) which is based on a functional connection between the additional fuel consumption and compensation of delay in which the combination of x_F and x_T can match any value in the operating cost index limits, model (14) and (15) is provided to minimize the costs caused by delays achieved by searching all possible combinations of altitude and that the cost index. values of fuel consumption and flight duration corresponding to each of these combinations.

First in a series of constraints ensures that selects only one of 25 possible combinations (5 and 5-level flight cost index value), and the objective function ensures that chooses the combination that gives the least cost. It can be seen that the objective function (14) and unemployment status values Δx_{Fij} and Δx_{Tij} instead e x_F and x_T values, which are used in the basic model. The basic model e x_F and x_T representing additional fuel consumption and the size of the delay that is compensated so they could have only positive values. Given that the model in this example has no such restrictions, but Δx_{Fij} and Δx_{Tij} and the difference compared to the reference case, changing the notation, and a x_{Fij} and x_{Tij} now represent the values of fuel consumption and flight time for each combination (Table 2) . The second and third constraint in the set of relations (15) are the way to determine the relative values of flight duration and fuel consumption, which are presented Table 3.Next in the series of constraints is modified in relation to the model of Chapter 2 to fit the model presented here, while other sets of relationships represented possible values of certain parameters.

Further work will be presented the results obtained using the model (14) (15) in the case described in section 6.1, the use of Excel. Results will be presented and explained, and the meaning of some parameters will be discussed in the context of the basic model (11) (12) of Chapter 2.

The possibility of compensation delay (x_K *) and the dependence of fuel consumption values of the delay compensation (* $x_F = f(x_T)$ *)*

As shown in Fig 1. it can be seen, with increasing altitude and decreasing cost index decreased significantly and fuel consumption, so the flight level 290, the maximum cost index (100) 12 250 kg of fuel consumed, while, For example, the optimum flight level 370, the minimum cost index (0), 9800 kg of fuel consumed. It can be seen that with increasing altitude and decreasing the difference between the maximum and minimum duration of the flight (minimum and maximum cost index, respectively), so the flight level 290, this difference is 60 minutes, while at flight level 370, this difference is only 16.8 minutes.

Table 3 presents the relative fuel consumption and time of flight to a base case of the flight under consideration (and Δx_{Fij} and Δx_{Tij} in the model (14) (15)). Based on these tables can be created that would be a photo opportunity with the change in delay compensation cost index and altitude (x_F and x_T in the model (11) (12)) (Fig 2.).

As can be seen, the time delay compensation can be achieved by increasing the cost index, reducing the level of flight, or the simultaneous application of these changes. The maximum value of delay that can be compensated (x_K in the mathematical model of Chapter 3) in the case under consideration here is 15 minutes and achieved flight level 290 at the cost index value of 100. The maximum value in case of change of consumer price index without a change in altitude is 7.8 minutes.From Fig 2. one can see how much extra fuel consumption values for specific cost index, i.e. reimbursement delays and flight altitudes and in one case, the maximum compensation delay is 2250 pounds. In chapter 4 of this paper it was stated that the functioning of the models need to determine the functional dependence of the additional fuel consumption $x_F = f(x_T)$. This functional dependence of the flight level 290 is shown in Fig 3. It can be seen that the functional dependence of third-degree polynomial form of the coefficient of determination (R^2), which is 0.9964.

Although the coefficient of determination is very high, in this example, when determining the minimum cost of delays caused by the functional dependency and functional dependency for the other flight levels are used because of their determination when sufficient data is not used, and these relationships can not be considered fully reliable .

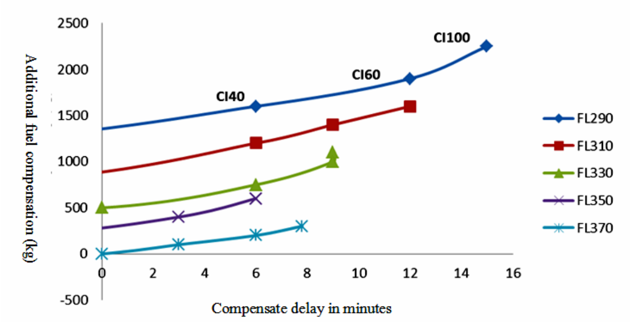

Fig 1. The functional dependence of the additional fuel consumption of compensation for different flight levels

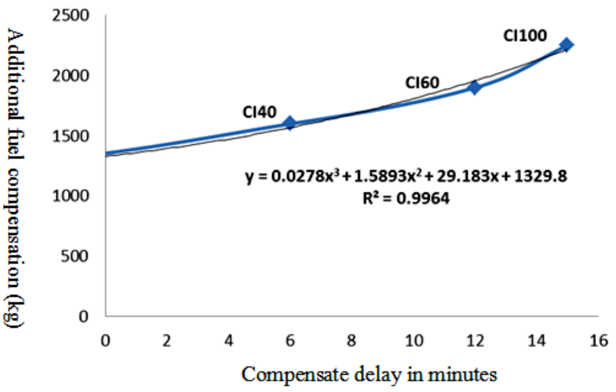

Fig 2. The functional dependence of the additional fuel consumption of compensation for delay FL290

3. Determination of minimum cost caused by delay

The basic assumptions of examples is said to be flying under normal conditions, the conditions without delay, carried out at flight level 370, the index of cost 20th If for some reason, which are discussed in the previous section of this paper is late taking off for 60 minutes, the flight performance of their normal flight parameters will lead to the same landing is delayed for 60 minutes (assuming no wind on a route or other sources of potential delays). To the airport of destination was delayed less than 60 minutes is necessary to change the flight parameters (FL and CI) (see Fig 2). In the case of unchanging parameters of the flight costs caused by the delay amount to EUR 4084.8, while these costs are only costs delays, because it does not consume extra fuel in compensating for the delay. minimal costs caused by flight delays are achieved at the level of 290 maximum cost index (100) and amounts to EUR 2759.4. Therefore, the minimum cost of delay caused in this case reaches the maximum possible compensation for the delay, which is 15 minutes. Therefore, the aircraft will arrive in this case the destination airport late, 45 minutes from now. From a total of EUR 2759.4, 697.5 are due to additional fuel consumption (2250 kg), and the remainder (EUR 2061.9) is caused by a delay of 45 minutes. In Fig 4. one can see the value of delay costs for each of the 25 possible combinations. At first glance one can see that the total cost decreases with the cost index, which is logical because the application of higher cost index reduces the delay, which is the base case is quite large. In doing so, it is logical, increasing the share of the additional fuel costs in total costs, because it uses extra fuel to compensate for the delay.

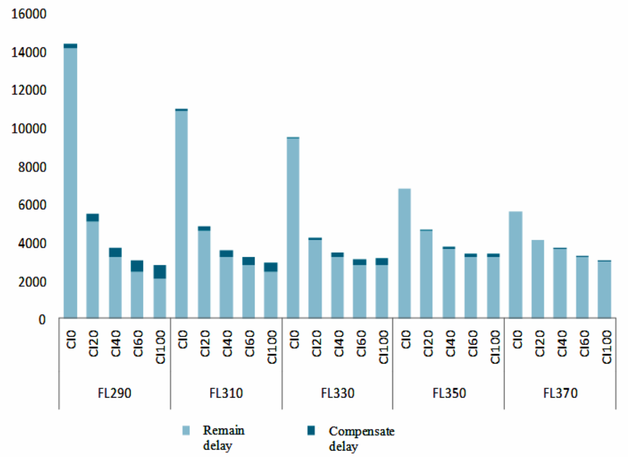

Fig 3. Flight cots induced by delay (starting delay at origin is 60min)

Notice also that the values of cost index values for the three costs is relatively similar for all flight levels, while significantly deviate cost index values for 20 and, above all, the cost index of 0th The reason is that we are using these small delay index can not compensate for that. Δx_{TK} s less than 0 (Fig 3.), so in these cases the costs are higher than costs in the original scenario.

4. Variation of different models of inputs

In order to better understand the effects of various parameters on the value of the minimum cost and delay caused by the index value of the cost and level flight for which this value is achieved, it is necessary to vary the model input data and observe the changes that happen in the output data of the model.

If you change the original delay, which is considered compensation, will change the value of the minimum costs caused by delays and flight data for the value that achieves the minimum cost. If the original 25-minute delay, the optimal solution will be cost flight index of 100, without a change in flight level (FL370). In this case, will be reimbursed the original 7.8 minute delay, so it will now amount to 17.2 minutes, with the total cost amount to 171.5 Euros.

The values of costs for the original delay of 25 minutes for all possible combinations of flight parameters are shown in Fig 4. It may be noted that the costs will be significantly lower than the costs in case of delay of 60 minutes, and make the most of the cost of additional fuel costs. As in the previous case, the highest costs in the case of small values of cost index. The reasons for this are mentioned in the previous section work. Change the value of the minimum cost function caused a delay with the change of original values of the delay is shown in Fig 5.

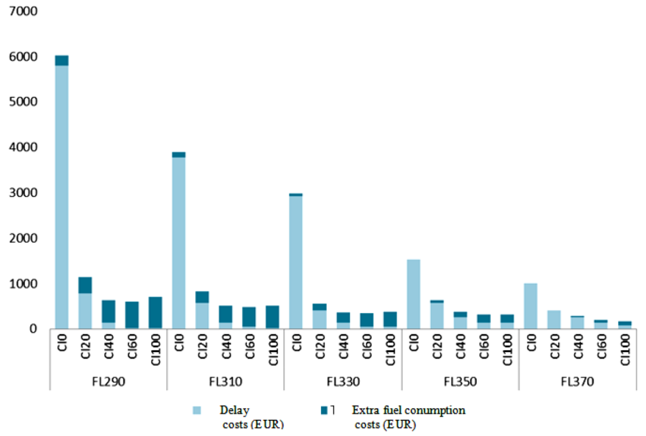

Fig 4. The flight cots induced by delay (starting delay at origin is 25min)

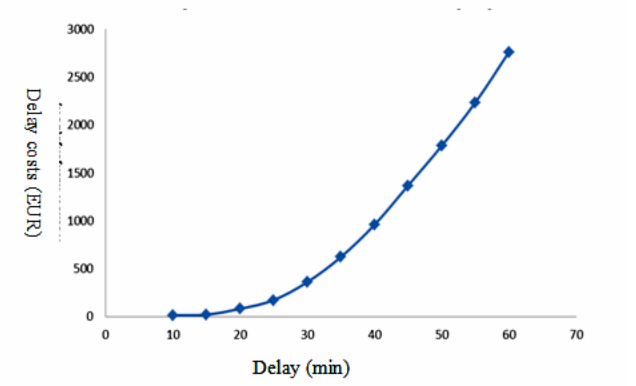

Fig 5. The function of minimal delay costs

The picture can be seen that this function has a relatively slight increase in the value of the original delay of 20-25 minutes, and after that the abrupt increase this value.

The logic of minimizing the costs caused by delays described in Chapter 3 of this work can be best understood by looking at Fig 5. In this Figure we show the values of the delay that should be recovered in certain cases, the original delay (10-60min) in order to achieve minimum costs, and flight parameters that correspond to the values of delay compensation, and these minimum costs. It is observed that is not accessed at all costs make up the maximum delay, but will make a balance between the compensation of delays and additional fuel consumption and recoup as much time as necessary to minimize the costs caused by delay, which is the goal of this model. It can be seen that in the case of large original delay (50-60min) to access the maximum compensation delay (15 minutes - FL290/CI100↓), and after that decreases the amount of compensation (FL370/CI100), to the value of the original delay of 15 minutes and less when it does not join up for the delay, because this would only lead to increased costs. Given the data presented in the previous chapter, it is interesting to see if it will make up a relatively small access the original delay in the case of higher unit costs of delay. This case could correspond to a situation where a flight that is considered a large number of transfer passengers, where every minute is important, so that we can arrive in time to the next flight. Fig 11. presents the function of unit cost of delay depending on the length of delay in case higher costs. The values of the costs of 16.6 Euros per minute for a delay of 15 minutes and 133.8 Euros per minute for a delay of 65 minutes set up are a linear function. When it is assumed that the minimum value of the unit cost is the same as in the case of delay of 15 minutes (16.6). For these higher values of unit costs of delay are again certain value cost function caused by the delay. The parameters of the flight, which is necessary to use the delay to be compensated in order to minimize costs for the original delay values in the range of 10 to 60 minutes, are shown in Fig 6. , it is evident that with increase in unit cost of delay increases the tendency for lower values of the original delays. It is necessary to minimize the access delay.

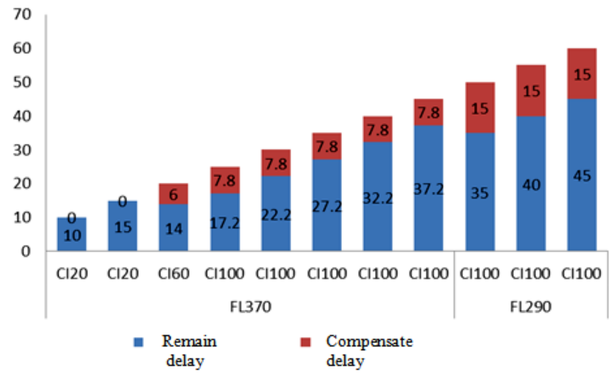

Fig 6. Compensation delay that gives the minimum flight costs caused by delay

Interestingly, in case the input value of 5 minutes late model gives a negative value of delay costs, which are carried on the flight FL370 0th index It can be concluded that in the case of given values of unit costs of delays and cost-effective fuel prices endure a delay of 14 minutes and save 200 pounds of fuel. Higher unit cost of delay to the original values of the delay of 30 minutes. It can also be seen that, unlike the base case, reducing the access delay in the case of the original small delay (15 minutes or less).It is expected that a reduction in unit cost of delay (as well as increasing the unit cost of fuel) in the opposite way affect the model outputs.

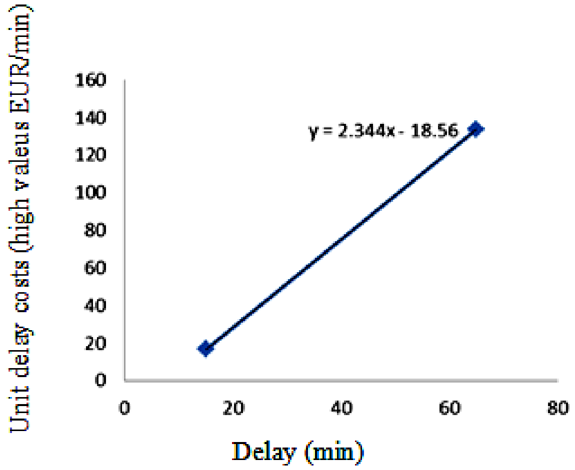

Fig 7.Unit delay costs (high values EUR/min)

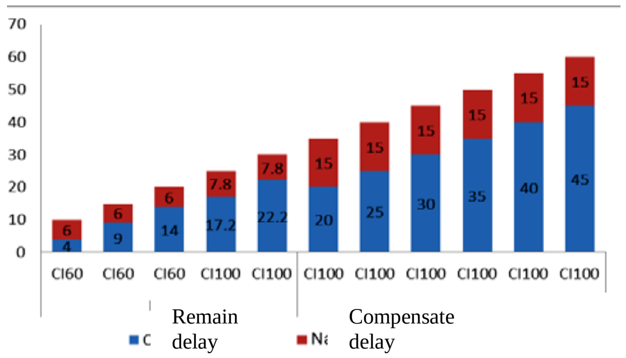

Fig 8. Compensation delay that gives the minimum cost of delays caused by (higher cost of delay)

5. Conclusion

This paper describes the concept of *dynamic cost index*. The *dynamic cost index* is the concept of sharing and collecting data that could produce minimal cost due to delays, by specifying the appropriate combination of flight cost and delay cost to compensating the delay (additional cost of trip fuel). In the introductory chapters were described terms of delays and *cost index*, which understanding is necessary for understanding the basic principles underlying the concept of *dynamic cost index*. After that we described each of the basic elements of this concept in particular. First, we explained that the company suffers the costs from a delay, then the role of air traffic control in this concept and the impact of aircraft performance. After that, the explanation of the negative impacts on the environment and their consideration within the concept, then a mechanism for exchange of data needed to calculate the minimal costs due to delays. The logic minimization of these costs is the simplified method described in a separate chapter, and then further explained by example.

In explaining the basic principles underlying the concept of dynamic cost index and its elements are not inquired into the specifics, but the goal was to review all potential impacts and connections between them. The reason lies primarily in the fact that many of these elements can vary from airline to airline, depending on the policy, the traffic network, fleet, etc... Therefore, this work primarily to serve as the basis for the airlines to develop their own means of determining the dynamic cost index in case of delays. The paper emphasizes the crucial fact that airlines have to consider when developing such tools:

- The *dynamic cost index* is a tool that is on the agenda. Tactical level helps companies to combat delays. Dealing with delays is done by determining the cost index, which should be applied, if known, the delay of the aircraft in order to minimize the marginal (additional) costs incurred due to delay times. This paper describes all the potential costs that must be taken into account and highlighted the fact that these costs are not fixed but will depend on the duration of the delay, as well as the specific flight (aircraft type, passenger occupancy ratio, profile and intentions of passengers, etc...). Also presented is the notion that we should take into account the transfer of the negative impact of delays to other flights of the company.
- Applying appropriate i.e. desired *cost index* in terms of delay performance of the aircraft is limited. In addition, this decision must be approved by ATC.
- You can not ignore the negative impacts of the additional fuel consumption on the environment.
- In order to properly determine the cost of a new index is needed to dispose of various constant data and for this purpose it is necessary to develop a suitable mechanism for data exchange.

Besides all the aforementioned, there are elements of this concept are not sufficiently explained, and which need further attention:

- The paper set out the facts about the practical use of different speeds to cruise in European airspace. It is necessary to further examine the possibility of changes in speed limits as a way of compensating the delay
- The paper mentioned the possibility of compensating the delay by changing the route. Such a possibility, but as mentioned is not considered further and the further development of this concept should pay more attention to this issue. Review and make up for this kind of delay is especially important if you are watching an example in section 6 of which can be seen that the maximum possible value of the delay can be compensated by a relatively small, especially on shorter routes. This method would probably require that the company has no information about possible alternative routes and additional fuel and time savings on these routes.
- It is necessary to further examine the effects of changes in flight profile change in *CI* (climb and descent).
- Is necessary to develop an appropriate procedure for evaluating the costs due to additional emissions.
- It is necessary to clearly identify changes in technology costs index, i.e. responsibilities of various actors in the concept of dynamic cost index.

In Chapter 3 an attempt was made to work on it more easily explain the logic of calculating the minimum cost of delay. The mathematical model presented in the above section should be simplified mathematical operations that are supposed to carry a tool (program, software) to determine the *dynamic cost index*. A simplified example of this analytical model, later developed in Excel, is presented in Chapter 4.The concept of *dynamic cost index* should help the airlines, who decide on its development in the tactical battle against the negative effects of delay. However, this concept requires great efforts of the company, which would be reflected in the hiring of new professionals and technical resources, as well as a complete reorganization of the airline's way of functioning, which is no easy task. However, given that the possibility of reducing delays by expanding airport capacity is rather limited, and that more companies have problems with the economic aspect of the business, it is certain that the airline will use all possible means to combat delays, so there is a possibility application of this concept by the same in the future.

REFERENECES

1. Cook A., Tanner G., Anderson S. (2004), "Evaluating the True Cost to Airlines of One Minute of Airborne or Ground Delay". University of Westminster for Performance Review Commission (EUROCONTROL)
2. Cook A., Tanner G., Williams V., Meise G. (2009), "Dynamic cost indexing – Managing airline delay costs", Journal of Air Transport Management, pp. 26-35.
3. Courdacher, T., & Mouillet, V. (2008). ADAPT Aircraft Data Aiming at Predicting the Trajectory: EUROCONTROL.
4. Flight Operations Support & Line Assistance (1998), "Getting to Grips With the Cost Index", Airbus
5. Green, S.M., Grace, M.P., & Williams D.H. (2000). Flight Test Results: CTAS and FMS Cruise/Descent Trajectory Prediction Accuracy. Proceedings of the 3rd USA/Europe Air Traffic Management R&D Seminar, Napoli, Italy. Klooster, J.K., Del Amo, A., & Manzi, P. (2009). Controlled Time-of-Arrival Flight Trials. Proceedings of the Eighth USA/Europe Air Traffic Management Research and Development Seminar (ATM2009), Napa California.
6. Mirosavljević P., Gvozdenović S., Čokorilo O., Lj. Vasov, The Transport Aircraft Dinamic Cost Index Determination Method:The Medium Range Routes, 15th Annual World Conference, *Air Transport Research Society (ATRS) World Conference*, Conference Proceedings CD issue, 20 pp., Sydney, Australia, July 2011.
7. Transport Studies Group, University of Westminster, London, (2008), "Dynamic Cost Indexing - Progress Report (Deliverable 1)", (EUROCONTROL)
8. Transport Studies Group, University of Westminster, London, (2008), "Dynamic Cost Indexing - Technical Discussion Document 4.2: Including climate impacts of NOx in a Dynamic Cost Indexing tool", (EUROCONTROL)
9. Transport Studies Group, University of Westminster, London, (2008), "Dynamic Cost Indexing - Technical Discussion Document 6.0: Airline costs of delayed passengers and how to estimate full network delay costs", (EUROCONTROL)
10. Transport Studies Group, University of Westminster, London, (2008), "Dynamic Cost Indexing - Technical Discussion Document 9.0: Aircraft maintenance – marginal delay costs", (EUROCONTROL)
11. Tysen Mueller, K., Bortins, R., Schleicher, D.R., Sweet, D.N., & Coppenbarger, R.A. (2004). Effect of Uncertainty on En Route Descent Advisor (EDA) Predictions. Proceedings of the AIAA 4th Aviation Technology, Integration and Operations (ATIO) Forum, Chicago, IL.
12. U.S. National Weather Service. (2008). NWS Instruction 10-806: World Area Forecast System.
13. Vietor, C.W. (1990). Precision Three-Dimensional Flying for Jet Transport Aircraft - The Profile Descent.
14. Xavier Prats and Menéndez (2011),Contributions to the optimisation of aircraft noise abatement procedures Doctoral Thesis, VIII abertis Prize - Honorable mention, Universitat Polithcnica de Catalunya

ACKNOWLEDGEMENT

The paper present part of research project ''Environment management system framework respect to aircraft engine emissions and risk of aircraft accidents around airports in Serbia'', supported by Ministry of Science and Technological Development of Republic of Serbia, number TR36001. The part of this research project, contains analyses of major air transport pollutants generation, such as CO_2, SO_2 and water vapour and NO_x CO, HC and their mitigation trough development of air pollution management, which apply on aircraft during taxi in, take-off, *en route* flight, landing flight operations and taxi out ground operation. The real application of air pollution management will support efforts of South East Europe Countries, Republic of Serbia and European Union in air transport pollution reduction process.