MATHEMATICAL MODEL FOR THE PREDICTION OF LIFE CYCLE COSTS OF LAND MILITARY VEHICLES

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Abstract:
The given mathematical model may be used for preliminary determination of life cycle costs of a new vehicle type or also for comparison of several vehicles of the same category. This is applicable especially for tenders where life cycle costs present an important criterion for the selection of a supplier. The article also contains a graphic interpretation comparing life cycle costs of Land Rover Defender, the values of which were gained from its operation, and values calculated with the help of the model draft for the prediction of life cycle costs.
KEY WORDS: *PREDICTIVE LIFE CYCLE COSTS, PREDICTIVE COSTS ON VEHICLE PURCHASE, PREDICTIVE COSTS ON OPERATING STATE, PREDICTIVE COSTS ON PREVENTIVE MAINTENANCE, PREDICTIVE COSTS ON CORRECTIVE MAINTENANCE.*

1. Introduction

At present, ensuring desired vehicle safety and minimal life cycle costs is required. At the same time it is necessary for the vehicles to operate without an inappropriate impact on the environment and road traffic. The decision on the vehicle purchase is influenced by not only initial (purchase) costs, but also by expected operational costs and maintenance costs for the whole vehicle durability (possession costs) and disposal costs. According to the given model vehicle suppliers could optimise an evaluate various strategies of operation, maintenance and disposal.
The life cycle costs analysis is an economic analysis process for assessing the total costs of purchase, possessing, and disposal of an item. The analysis may be used within the whole life cycle of the item, or in some parts, or in combinations of various periods of the life cycle [1].
The primary guidance for vehicle life cycle cost evaluation is a standard [1]. According to this standard the vehicle life cycle cost can be divided into these five periods:

1. period of concept and requirements determination,
2. design and development period,
3. manufacture period,
4. operating state and maintenance period,
5. disposal period.

2. Calculation of Predictive Life Cycle Costs of Combat Vehicles

Generally, the total costs expended during the given periods may be divided to purchase costs, possession costs, and disposal costs [1]. For the draft model, division of the life cycle costs to the following five categories is recommended.

$$LCC = C_P + C_{OMC} + C_{OMP} + C_{OMO} + C_D \tag{1}$$

where LCC - predictive life cycle costs of combat vehicles; C_P - costs on vehicles purchase; C_{OMC} - predictive costs on corrective maintenance; C_{OMP} - predictive costs on preventive maintenance; C_{OMO} – predictive costs on operating state of vehicles; C_D - predictive costs on liquidation of combat vehicles.

2.1 Calculation of costs on vehicles purchase

The vehicle purchase cost can be expressed by the following equation [1]:

$$C_P = C_{CD} + C_{DD} + C_M + C_S + C_G \tag{2}$$

where C_{CD} - costs on the period of concept and requirements determination; C_{DD} - costs on the design and development period; C_M - costs on the manufacture period; C_S - costs on the vehicle sale period; C_G - costs on ensuring repairs during a guarantee period.

2.2 Costs on amortization

The actual value of a vehicle during its operation shall be calculated from costs on vehicle purchase reduced by its amortization. Costs on amortization include the vehicle age and kilometrage.
The value of amortized vehicle shall be determined upon the vehicle operating time (age) and mileage. For a certain vehicle type, the price shall be calculated from amortization scales, in which a basic percentage deduction for the operating time and a basic percentage deduction for mileage are determined. The vehicle value shall then be calculated as an arithmetic average of the following values:

$$C_{PA} = \left(C_{AT} + C_{AO}\right)/\,2, \tag{3}$$

where: C_{PA} - costs on vehicle purchase and amortization, C_{AT} - amortization value of the vehicle depending on the operating time, C_{AO} - amortization value of the vehicle depending on its mileage.

2.3 Calculation of predictive costs on vehicles maintenance

The total vehicle maintenance costs comprise of preventive maintenance costs and corrective maintenance costs.

$$C_{OM} = C_{OMC} + C_{OMP} \tag{4}$$

The maintenance costs comprise of material costs, labour costs, and workshop equipment costs [1].

$$C_{OM} = (C_{OMCM} + C_{OMCL} + C_{OMCF}) + (C_{OMPM} + C_{OMPL} + C_{OMPF}) \tag{5}$$

where C_{OM} - maintenance costs; C_{OMC} - corrective maintenance costs; C_{OMP} - preventive maintenance costs; C_{OMCM} - costs of material used for corrective maintenance; C_{OMCL} - costs of labour force for corrective maintenance; C_{OMCF} - costs of workshop equipment used for corrective maintenance; C_{OMPM} - costs of material used for preventive maintenance; C_{OMPL} - costs of labour force for preventive maintenance; C_{OMPF} - costs of workshop equipment used for preventive maintenance.

1) Costs prediction on corrective maintenance

The total costs which are required for ensuring repairs during the vehicle operating time depend on the number of failures which occur in the vehicle during its operation, and on costs necessary for removing these failures. Corrective maintenance costs may be calculated as follows (6), (7) a (8), [3] [4]:

$$C_{OMC_{(j)}} = \sum_{n=1}^{n=j} z_{(t)} \cdot ic_R \tag{6}$$

$$f_{C_{OMC}} = c_R \cdot \int_{t_0}^{t_n} \frac{t}{E_t} \,\mathrm{d}t \tag{7}$$

$$C_{OMC} = \frac{c_R}{\phi} \cdot t \tag{8}$$

$$z(t) = \lim_{\Delta t \to 0+} \frac{E[N(t+\Delta t) - N(t)]}{\Delta t} \tag{9}$$

$$E(t)=\alpha \cdot K_{\beta}=\alpha \cdot \Gamma \cdot (1+\frac{1}{\beta}) \quad (10)$$

where: $C_{OMC_{(j)}}$ - prediction of costs on corrective maintenance during operating time; t - operating time in kilometres; i - determined value of the interval in kilometres; j - number of determined intervals i; $z_{(t)}$ - failure intensity in interval t; $E_{(t)}$ - mean time between failures depending on mileage , calculated with Waybill distribution; α, β – parameter Waybill distribution; Γ – gamma function of Waybill distribution; $f_{C_{OMC}}$ - behaviour of the function expressing costs of corrective maintenance during operating time t; Φ - mean time between failures; c_R - average cost on one failure repair, consisting of costs on material and costs on work.

The calculation of prediction of corrective maintenance costs shall be performed using at least one of the following methods, depending on information availability [3] [4]:

a) from the calculation of failure intensity $z_{(t)}$,
b) from the calculation of the mean time between failures $E_{(t)}$ upon Waybill distribution and using Nelson's method [5] [6],
c) from the calculation of the mean time between failures c_R – so called common mean time which does not include the process of degradation.

ad a) Calculation of failure intensity

Failure intensity $z(t)$ is the limit of a ratio – if it exists – of the mean failure number of a repaired item within $(t, t + \Delta t)$ time interval to the length of this interval Δt, if the length of the time interval is approaching zero.

$$z(t) = \lim_{\Delta t \to 0+} \frac{E[N(t+\Delta t) - N(t)]}{\Delta t}, \quad (11)$$

where $N(t)$ - number of failures within $(0, t)$ time interval, E - expected value, $t + \Delta t$ - time interval.

Using the given expression, it is possible to calculate failure intensity for the whole vehicle or for individual groups of the vehicles.

ad b) Calculation of the mean time between failure $E_{(t)}$ upon Waybill distribution and using Nelson's method

A number of machine parts and other equipment, to which exponential distribution does not apply, have durability time (operating time between failures) of Waybill distribution. Especially those machine parts showing mechanical wear and material fatigue. Also mechanical properties of materials, for example strength, have Waybill distribution.

Then it is necessary:

A) Determination of parameters α, β for Waybill distribution by Nelson's method

This is one of methods suitable for processing generally censor files. The estimation of parameters is based on the linearization of a relation for cumulative failure rate $H(x)$. This solution applies to the *Waybill distribution*. Distribution function of two-parameter W-distribution is given by the following relation [5] [6]:

$$F_x(x) = 1 - \exp\left[- H(x)\right], \quad (12)$$

where $H(x)$ is cumulative failure rate given for W-2 distribution by the following relation:

$$H(x) = \int_0^x \lambda(t)\, \mathrm{d}t = \int_0^x \frac{b}{\theta} \left(\frac{t}{\theta}\right)^{b-1} \mathrm{d}t, \quad (13)$$

Application of logarithm on (12) will result in the following equation, linear between parameters:

$$\ln H(x) = b \ln x - b \ln \theta. \quad (14)$$

The equation is already linear between parameters and may be solved in various ways:

a) graphically on an appropriate logarithmic paper,
b) numerically (e.g. by least squares method) [6].

B) Determination of confidence interval for parameters α with known parameters β and γ

Two-sided confidence interval $\langle a_D, a_H \rangle$ for parameter α with known parameters β, γ with confidence coefficient 1 - α shall be determined.

$$a_D = \left(\frac{2.\sum_{i=1}^{n} (x_i - \gamma)^{\beta}}{\chi^2_{1-\frac{\alpha}{2}}(2n)} \right)^{\frac{1}{\beta}}, \quad (15)$$

$$a_H = \left(\frac{2.\sum_{i=1}^{n} (x_i - \gamma)^{\beta}}{\chi^2_{\frac{\alpha}{2}}(2n)} \right)^{\frac{1}{\beta}}. \quad (16)$$

C) Mean Time between Failures Calculated with the Waybill Distribution

The calculation of mean time between failures for a vehicle (or groups) may be performed in longer intervals (e.g. 50 000 km). After determining values in these intervals, curve regression with function expression may be carried out. In this way the values of mean time between failures in the interval from production to disposal may be calculated.

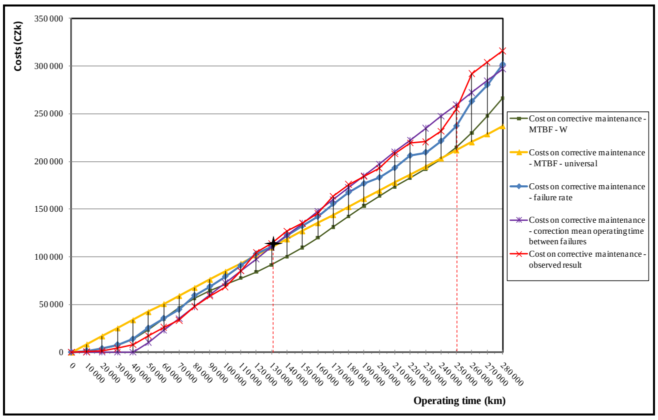
The calculation shall be done in each interval using the following procedure [2]:

1. Failures according to consequences are selected and divided to intervals; calculation includes also unclosed file values when a failure did not occur.
2. Using Nelson's method, cumulative rate $H(x)$ values are calculated.
3. Using Waybill diagram, cumulative values $H(x)$ are plotted depending on the kilometrage.
4. Values α and β are read from the Waybill diagram.
5. Mean time between failures is calculated using the Waybill distribution and calculated values α and β.
6. Resulting values are plotted in the diagram and regression curve with function expression is calculated. Upon theoretical as well as practical knowledge it is recommended to employ regression analysis. In this analysis, special non-linearizable models shall be used with exponential trend. The most appropriate particular regression curve has the following general form:

   $$y = b_1 e^{-\beta_1 x} + b_2 e^{-\beta_2 x}, \quad (17)$$

   where y - mean time between failures in kilometres, x - operating time in kilometres, b_1, b_2 - estimation of parameters, β_1, β_2 - estimation of parameters.
7. Mean time between failures in defined maximum durability time of a vehicle is calculated from the given equation.
8. Confidence interval for parameter α with known parameter β is determined.

Upon the given equations (6), (7) and (8), the corrective maintenance cumulative costs shall be calculated for the monitored predicted interval. Calculations of corrective maintenance costs, which are performed upon the failure intensity and mean time between failures upon Waybill distribution, include the process of aging and wear (degradation of the vehicle as a whole). On the contrary, calculation based on the mean time between failures, determined as an average distance between individual failures, does not include effects of degradation. Therefore, utilization of so called degradation coefficient is proposed, the aim of which is to implicate aging and wear of the vehicle. This calculation of the degradation coefficient is based on the calculation of corrective maintenance costs upon the mean time between failures – so called common meantime, and on actual monitoring of costs for one vehicle type of a given category [3].



Graph 1: Comparison of calculation costs on corrective maintenance of Land Rover Defender vehicles [3] [4]

2) Costs prediction on preventive maintenance

The costs given include costs of preventive maintenance which is performed in compliance with a specified schedule for maintenance of a given vehicle. The total amount of costs which shall be expended on ensuring the preventive maintenance during the vehicle operation depends on the number of preventive maintenance actions (maintenance interval) performed on the vehicle during its operation. The amount of these costs further depends on price relations of preventive maintenance actions, comprising of material price and labour costs. Thus, for costs of ensuring preventive maintenance it may be written [2]:

$$C_{OMP} = t \cdot \hat{c}_M \tag{18}$$

where C_{OMP} - predictive costs on ensuring preventive maintenance during operating time *t; t* - operating time in kilometres; $\hat{c}_M$ - average cost on ensuring preventive maintenance, consisting of costs on material and costs on work relating to an operation time unit.

For the calculation of preventive maintenance costs prediction the following values are selected [3] [4]:

a) service maintenance interval (km),
b) frequency of service maintenance during the service maintenance period,
c) types of service maintenance during the service maintenance period,

From these values, the following indicators shall be calculated:

a) guarantee inspection costs (price),
b) costs of individual types of service maintenance, comprising of material price and labour costs (price),
c) costs of the service maintenance period (price),
d) average costs of service maintenance relating to a kilometre of operation (price/km),
e) cumulative costs of service maintenance during the operation time (price),
f) elaborated regression curve with the expression of the most suitable function. Upon theoretical and practical knowledge, this will be mostly linear regression.

2.4 Costs prediction on vehicles operation

The period of operation includes fuel costs C_F, costs of service fluids, oils and lubricants $C_{OL,}$ which are refilled during the operation (not within service maintenance), tyre costs C_T, accumulator battery costs C_{AB}, costs of the vehicle insurance and road tax, and other possible costs resulting from the legislation C_{IRT}, motorway sticker costs C_{MT}, costs of technical condition control C_{TC}, exhaust-emission measurement costs C_E [3] [4].

$$C_{OMO} = C_F + C_{OL} + C_T + C_{AB} + C_{IRT} + C_{MT} + C_{TC} + C_E \tag{19}$$

$$C_F = \frac{c_{aF}}{100} \cdot p_F \cdot t_0 \tag{20}$$

$$C_{OL} = \frac{c_{aOL}}{100} \cdot p_{OL} \cdot t_0 \tag{21}$$

$$C_T = \frac{t_0}{d_{aT}} \cdot n_T \cdot p_T \tag{22}$$

$$C_{AB} = \frac{t_0}{d_{aAB}} \cdot n_{AB} \cdot p_{AB} \tag{23}$$

$$C_{IRT} = C_I + C_{RT} \tag{24}$$

where C_{OMO} – predictive costs on operation; c_{aF} - average fuel consumption (l/100 km); p_F - price per a litre of fuel (price/l); t_o - operation time (km); c_{aOL} - average consumption of oil and lubricant (l/100 km); p_{OL} - oil and lubricant price (price/l); d_{aT} - average tyre durability (km); n_T - number of tyres on the vehicle (pcs); p_T - tyre price (price); d_{aAB} - average accumulator battery durability (km); n_{AB} - number of accumulator batteries in the vehicle (pcs); p_{AB} - accumulator battery price (price); C_I - vehicle insurance price (price); C_{RT} - road tax price (price).

For the prediction of operational costs the following data must be available [3] [4]:

a) price per a litre of fuel (price),
b) average consumption of the vehicle in litres (l/100 km),
c) accumulator battery price (price),
d) accumulator battery durability in kilometres (years),
e) tyre price (price),
f) tyre durability (km),
g) price of annual liability insurance (price),
h) price of annual accident insurance (price),
i) price of annual motorway sticker (price),
j) costs of technical inspection and exhaust-emission measurement per year (price).

Upon specified data and with using equations (19–24), the total cumulative operation costs shall be calculated [3].

2.5 Costs prediction on disposal vehicles

This category includes costs of putting out of operation and disposal of vehicles with terminated durability.

$$C_D = C_{DD} + C_{DR} \tag{25}$$

where C_{DD} - costs of dismounting and removing engineering parts; C_{DR} - costs of recycling or safe disposal.

These disposal costs may present as a plus or minus value, depending on the disposal method. The plus value may be achieved if the vehicle is exploited and individual raw materials handed over to refuse collection. The minus values will be achieved if the vehicle is ecologically disposed of by another company.

A law including a vehicle disposal method as a duty of the manufacturer is being considered. This price would be included in the vehicle price, as it is for example in electrical appliances.

3. Results and discussion

In the following table are presented input value, which we must know, so that we could predict the costs of the life cycle vehicle. Input costs are presented in the third column which they were in the time treatment of work actual. Supporting calculated values are introduced in fourth column which we must calculate.

Costs on preventive maintenance, corrective maintenance, costs on purchase and amortization vehicle and operating costs are possible calculate on the basis of the assembled program. It is possible calculate on the basis of the introduced source date including the costs of the life cycle vehicles. In this case we can model to the operating time 1 000 000 km, which it is the expected lifetime period at this time the used vehicles. There is performed the calculation until at operating time 250 000 km because this point

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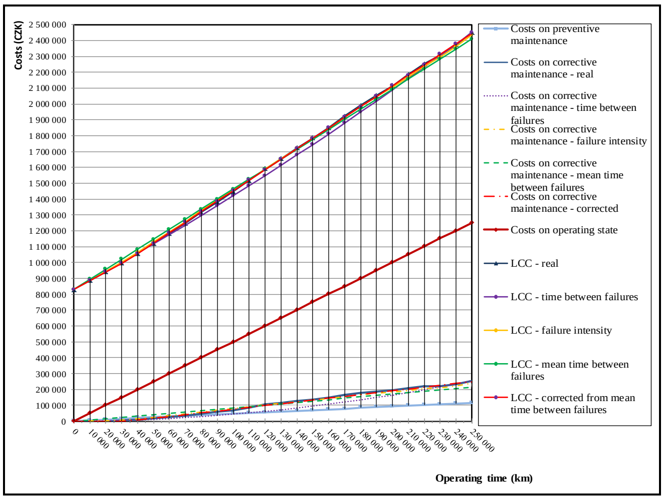
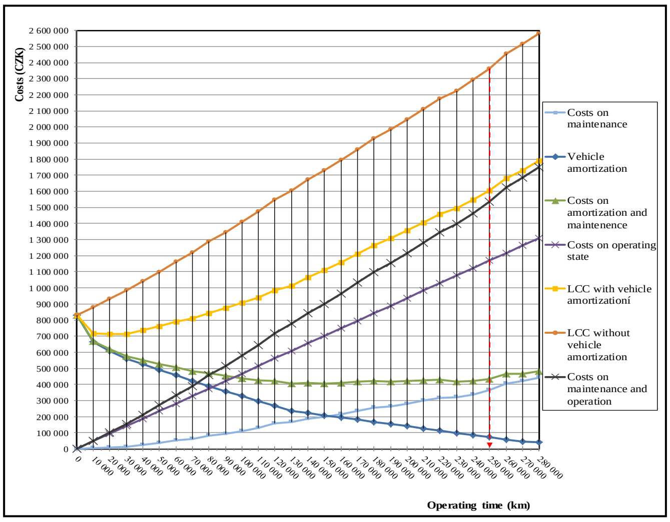
was determined as an optimum point for disposal Land Rover Defender vehicles from using.

Table 1: Calculation of life cycle costs of Land Rover Defender vehicles [3] [4].

Item name	Unit	Input values	Calculated values
Basic values			
Purchase costs	(CZK)	830 000	-
Durability time	(km)	250 000	-
Durability time	(yers)	25	-
Year average operating time	(km)	10 000	-
Preventive maintenance (PM)			
Service maintenance interval	(km)	10 000	-
Period of total service maintenance interval	(km)	40 000	-
Number of service maintenance during period	(-)	4	-
Costs of guarantee inspection	(CZK)	1 313	-
Costs of SM-I	(CZK)	2 311	-
Costs of SM-II	(CZK)	6 208	-
Costs of SM-III	(CZK)	7 317	-
Costs of total period SM	(CZK)	-	*18 146*
Average costs of PM of 1 km	***(CZK/km)***	-	***0,454***
Corrective maintenance (CM)			
Costs of 1 failure	(CZK)	4 300	-
Average mean time between failures	(km)	5 100	-
Average costs of 1 km	*(CZK)*	-	*0,85*
Failure intensity	-	-	*Graph 2*
Costs of CM depending on failure intensity	*(CZK/km)*	-	*Graph 2*
Costs of CM depending on mean tiume between failures - Weibull distribution	*(CZK)*	-	*Graph 2*
Costs of CM with correction MTBF - general	*(CZK)*	-	*Graph 2*
Costs of vehicle operation			
Average fuel consuption	(l/100 km)	10	-
Price per a litre of fuel	(CZK)	30	-
Operation time	(km)	250 000	-
Average consumption of oil and lubricant	(l/100 km)	0,1	-
Oil and lubricant price	(CZK)	100	-
Average tyre durability	(km)	40 000	-
Number of tyres on the vehicle	(piece)	4	-
Tyre price	(CZK)	3 600	-
Average accumulator battery durability	(yers)	4	-
Number of accumulator batteries in the vehicle	(piece)	1	-
Accumulator battery price	(CZK)	1 700	-
Vehicle insurance price	(CZK)	8 300	-
Road tax price	(CZK)	0	-
Pprice of annual motorway sticker	(CZK)	900	-
Costs of technical inspection per year	(CZK)	350	-
Costs of exhaust-emission measurement per ye	(CZK)	300	-
Total operation costs of 1 km	***(CZK/km)***	-	***4,7***
Liquidation costs	***(CZK)***	***0***	-



Graph 2: Comparison of calculation predictive and real life cycle costs of Land Rover Defender vehicles [3] [4].



Graph 3: Calculation life cycle costs of Land Rover Defender vehicles.

4. Conclusion

The article describes a model for the prediction of vehicle life cycle costs. It contains formulas, procedures, and necessary data for the calculation of individual costs of the vehicle life cycle. The method described may be applied to the preliminary determination of life cycle costs of a new vehicle and to a comparison of several vehicles of the same category. This comparison may be applied as a criterion in tenders for the supply of new technical equipment.

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